Heat transfer

$$q_x'' = -\lambda_x \frac{d\theta}{dx}$$

thermal conductivity [W/(mK)]

heat flux [W/m^2]

Energia armazenada = Energia entra - Energia sai + Energia gerada

$$\rho c \frac{\partial \theta}{\partial t} = \lambda_x \frac{\partial^2 \theta}{\partial x^2} + \lambda_y \frac{\partial^2 \theta}{\partial y^2} + \lambda_z \frac{\partial^2 \theta}{\partial z^2} + q'''$$

no internal heat sources

$$\frac{\partial \theta}{\partial t} = \alpha_x \frac{\partial^2 \theta}{\partial x^2} + \alpha_y \frac{\partial^2 \theta}{\partial y^2} + \alpha_z \frac{\partial^2 \theta}{\partial z^2}$$

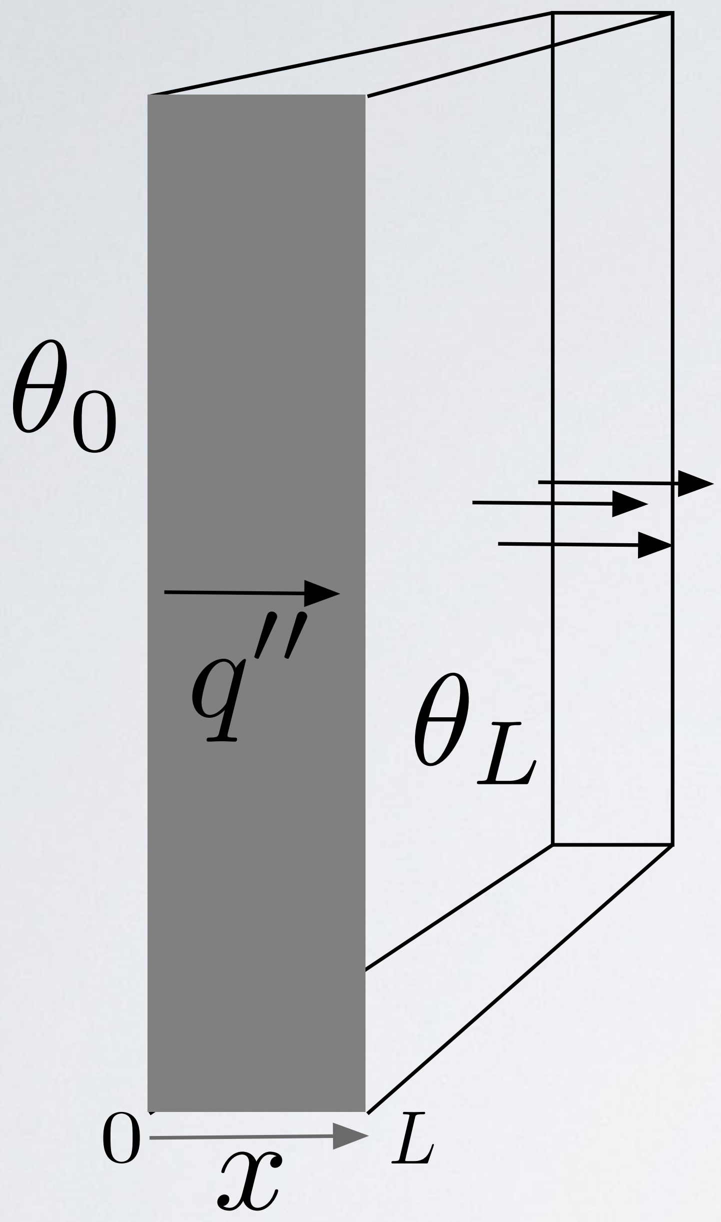
Thermal diffusivity [m^2/s]

$$\alpha \equiv \frac{\lambda}{\rho c}$$

$$\frac{\partial \theta}{\partial t} = \alpha_x \frac{\partial^2 \theta}{\partial x^2} + \alpha_y \frac{\partial^2 \theta}{\partial y^2} + \alpha_z \frac{\partial^2 \theta}{\partial z^2}$$

Steady-state $\frac{\partial \theta}{\partial t} = 0$

Transient $\frac{\partial \theta}{\partial t} \neq 0$



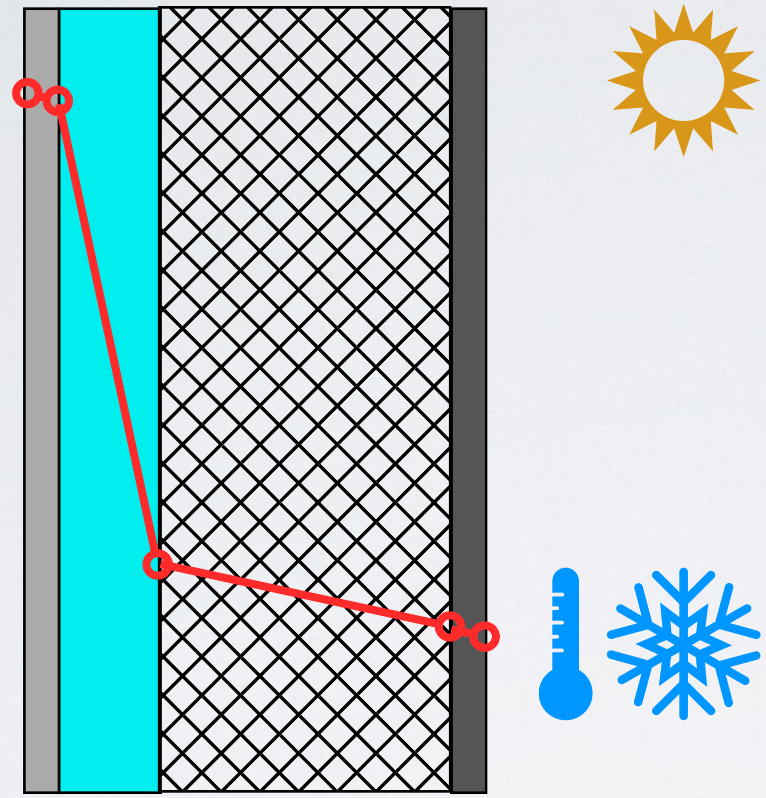
$$\frac{\partial^2 \theta}{\partial x^2} = 0$$

$$q'' = \frac{\lambda}{L}(\theta_0 - \theta_L)$$

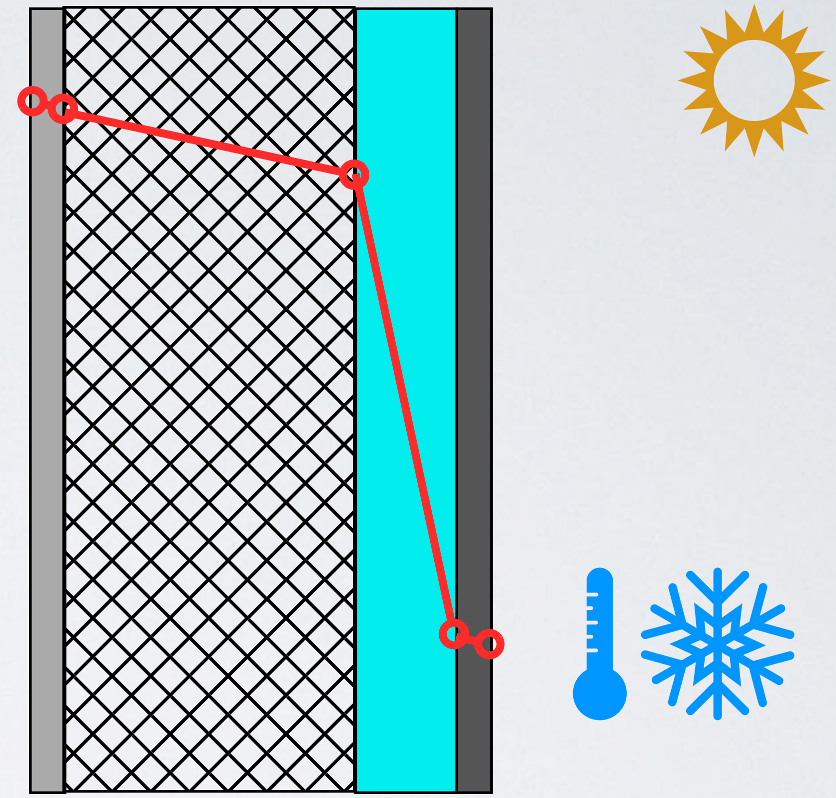
$$R'' = \frac{L}{\lambda}$$ [m²K/W]

$$R = \frac{L}{\lambda A}$$ [K/W]

internal insulation



external insulation



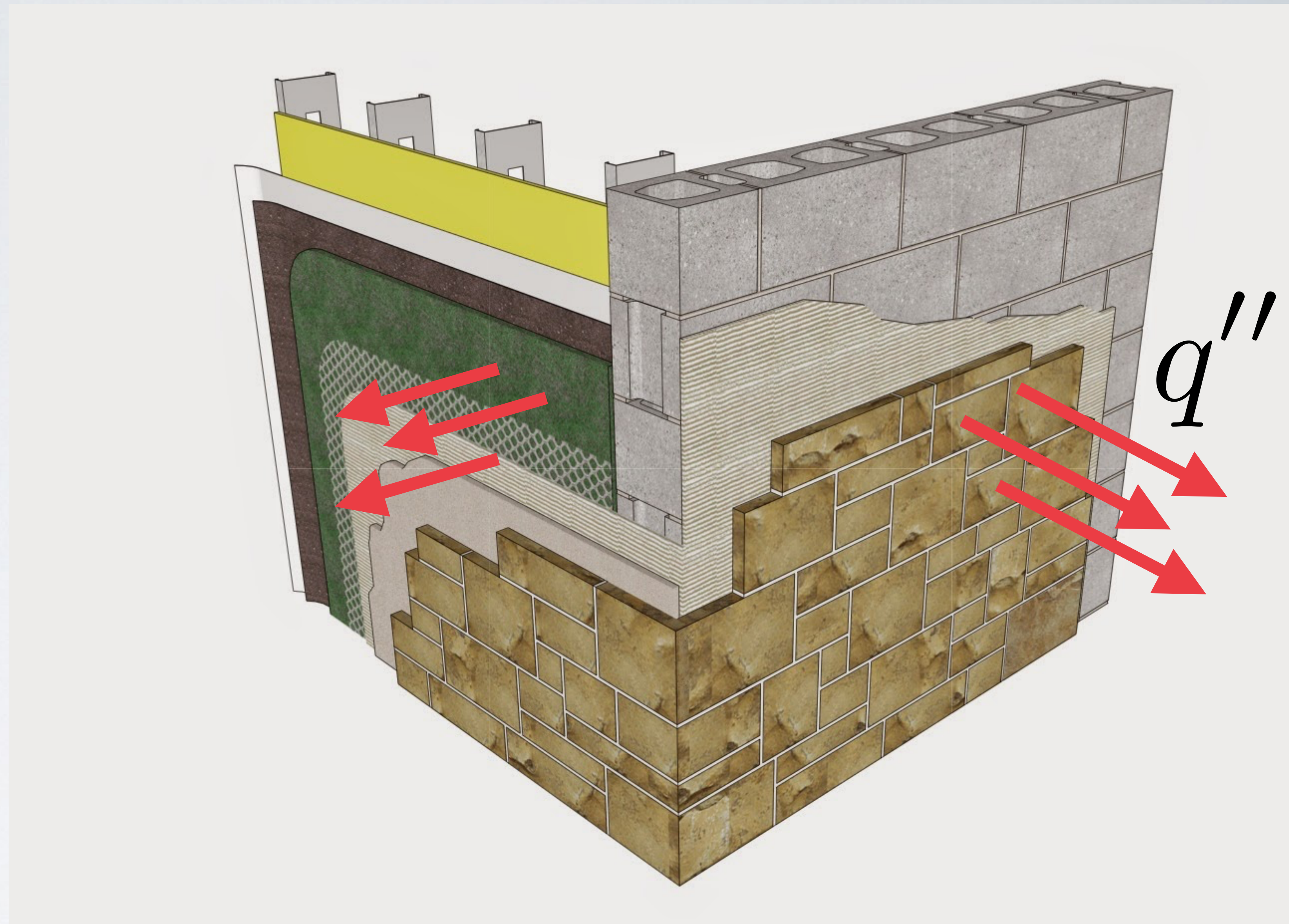
$$q'' = \frac{\Delta\theta}{R''} \qquad \Delta\theta = R'' q''$$

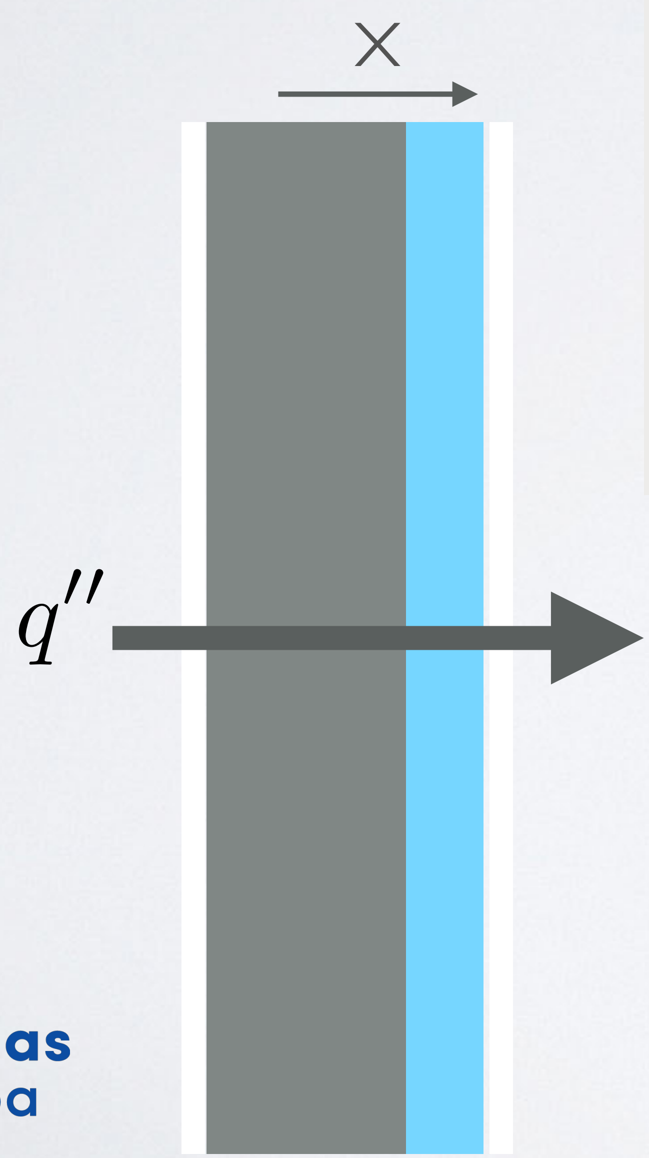
One-dimensional heat flow

Steady-state conditions

Multiple layers

$$\frac{\partial^2 \theta}{\partial x^2} = 0$$





$$U = \frac{1}{R''_{eq}}$$ → includes surface resistance

$$U = \frac{q^*}{A\Delta\theta}$$

Ciências ULisboa

Serie

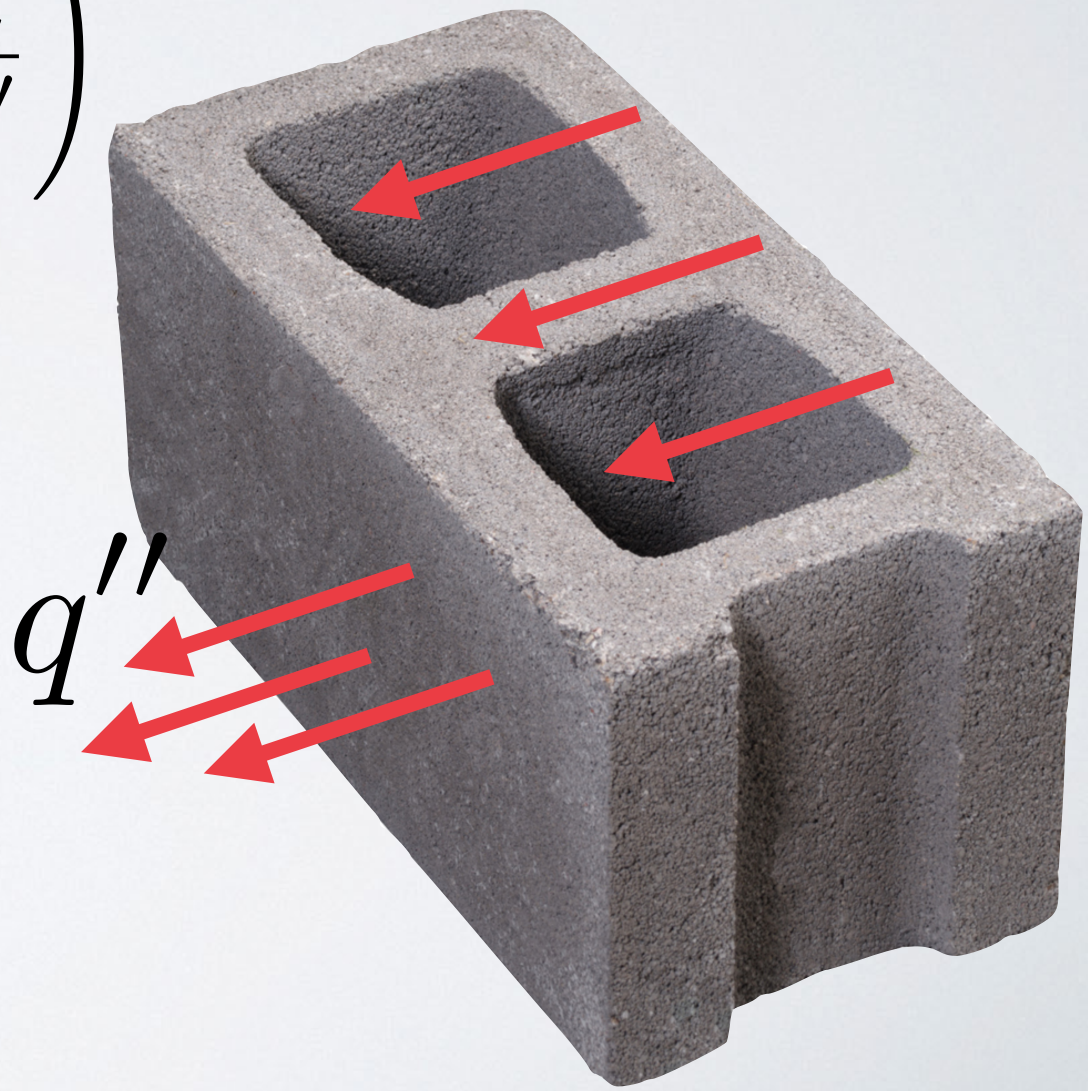
$$R_{eq} = \sum_i R_i$$

$$R''_{eq} = \sum_i R''_i$$

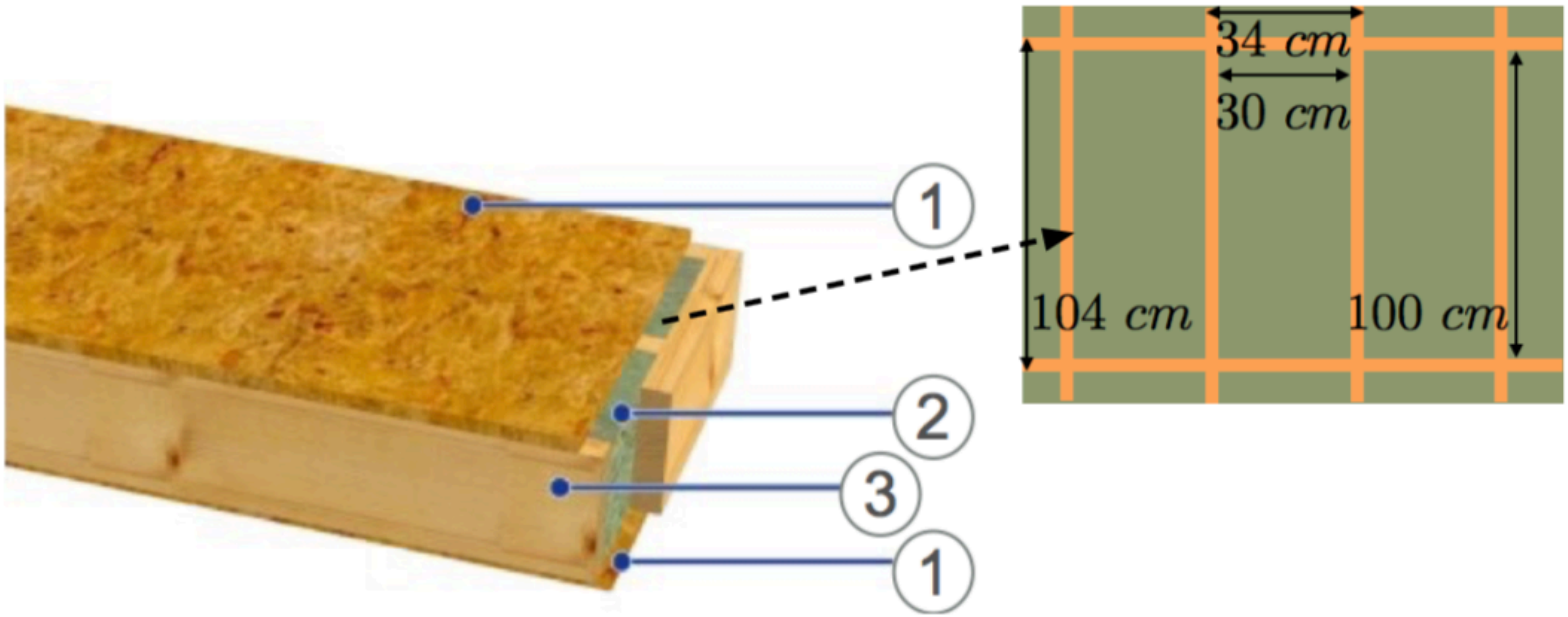
Parallel

$$R_{eq} = \left(\sum_i \frac{1}{R_i}\right)^{-1}$$

$$R''_{eq} = \left(\frac{1}{A}\sum_i \frac{A_i}{R''_i}\right)^{-1}$$

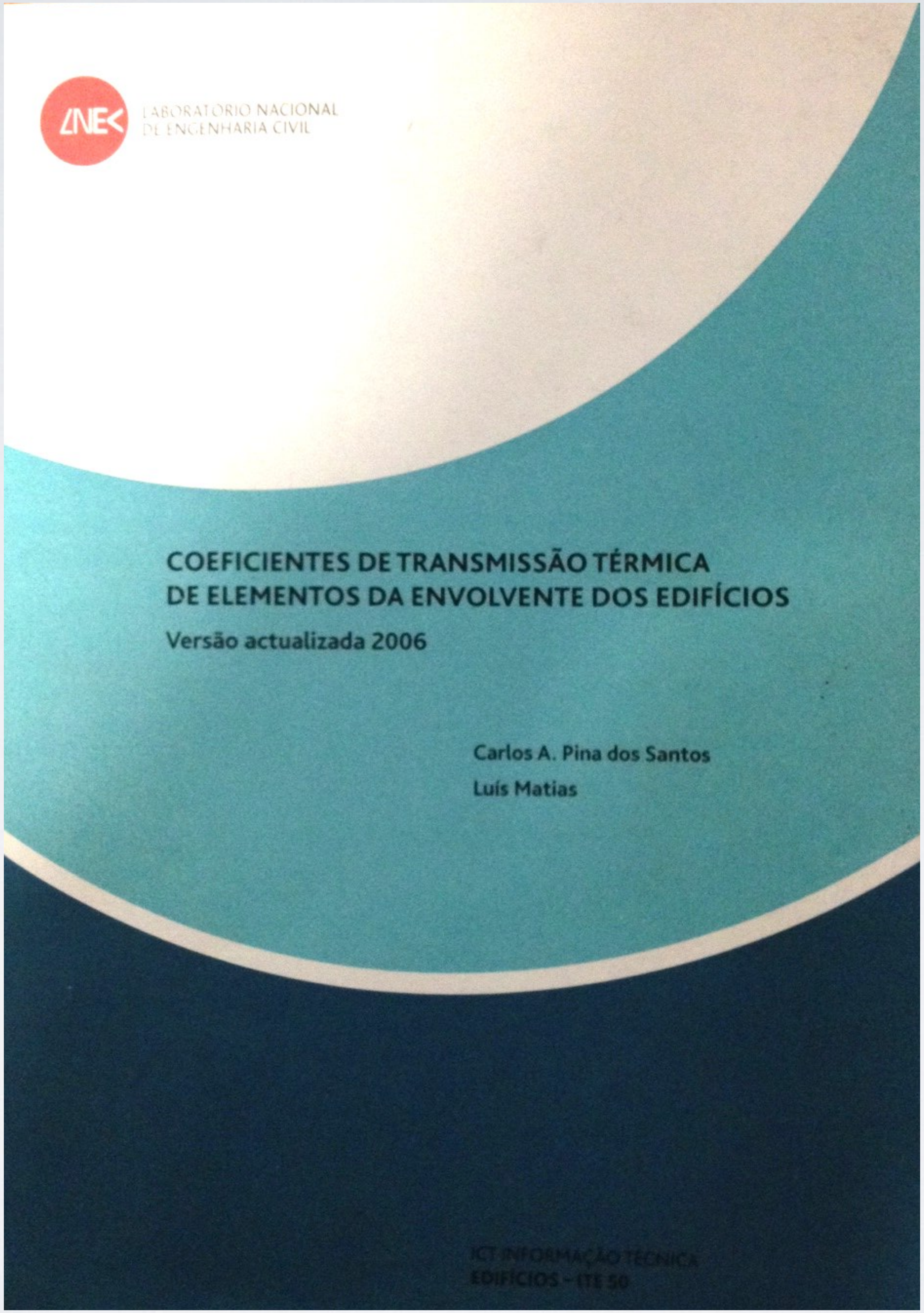


Exercício 1.1.7 **Calcular o coeficiente de transmissão térmica superficial** (U) do pavimento da figura, considerando que as resistências térmicas superficiais são, para ambas as superfícies, $0.17\ W/(m^2K)$.



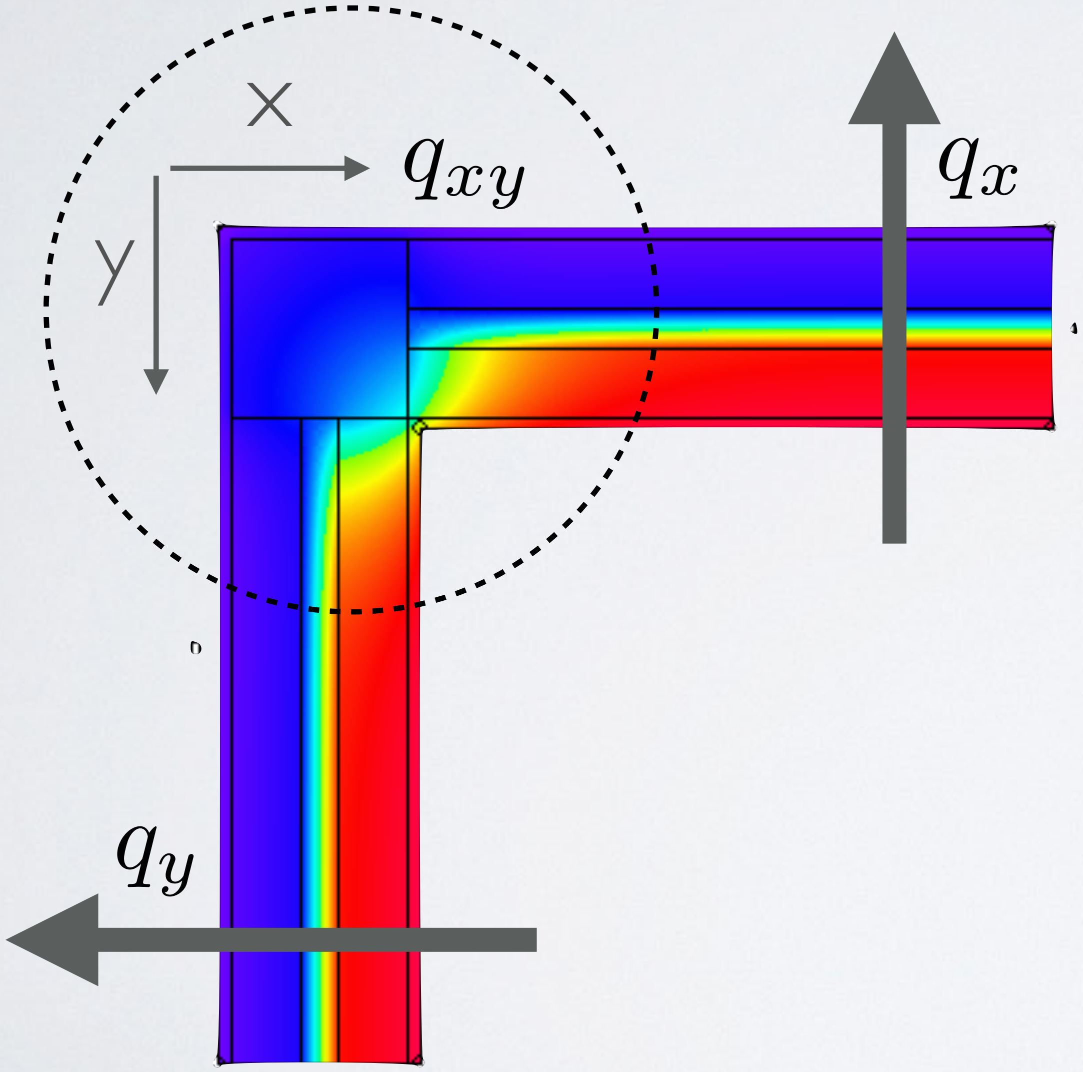
Materiais:

1. Painéis OSB: 2.2 cm de espessura, $\lambda = 0.13\ W/(mK)$
2. Lã mineral: 10 cm de espessura, $\lambda = 0.04\ W/(mK)$
3. Barrotes de madeira: 10 cm de espessura, $\lambda = 0.23 W/(mK)$


LABORATÓRIO NACIONAL
DE ENGENHARIA CIVIL
COEFICIENTES DE TRANSMISSÃO TÉRMICA
DE ELEMENTOS DA ENVOLVENTE DOS EDIFÍCIOS
Versão actualizada 2006
Carlos A. Pina dos Santos
Luís Matias

Two-dimensional heat flow
Steady-state conditions
Multiple layers

$$\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} = 0$$



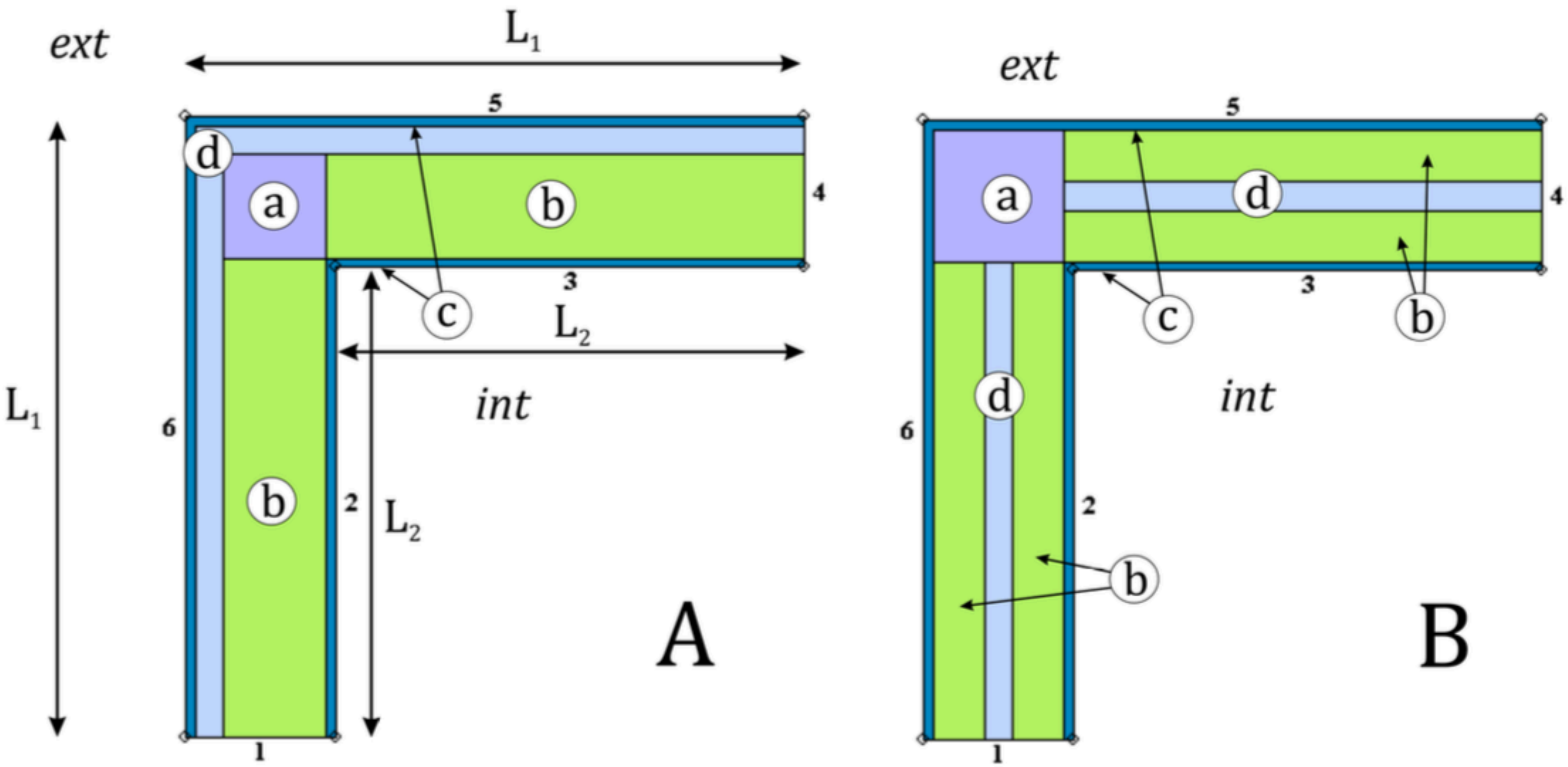
$$q^* = q_x + q_y + q_{xy}$$

$$q^* = (A_x U_x + A_y U_y + \Psi B)\Delta\theta$$

$$\Psi = \frac{q^*/\Delta\theta - A_x U_x - A_y U_y}{B}$$

Two walls connection

Exercício 1.1.5 Considerar os pormenores construtivos A e B (vista em planta) que representam a ligação entre duas paredes de fachada ligadas por um pilar de betão (elemento a). A altura total da fachada (medida pelo interior) é de 3 m. As superfícies 1 e 4 da figura são adiabáticas. As condições exteriores (*ext*) e interiores (*int*) encontram-se representadas na figura.

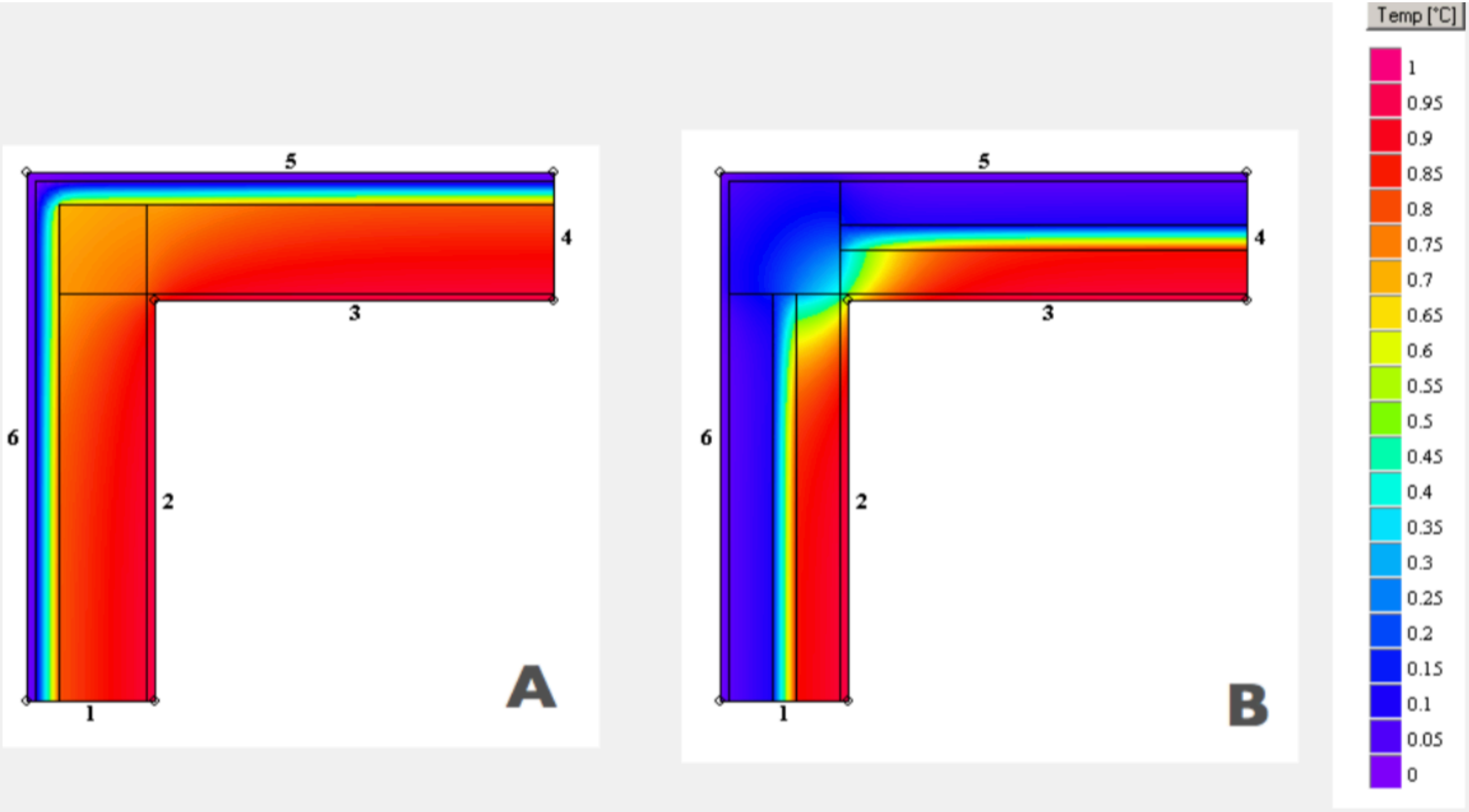


Dimensões: $L_1 = 1.3\ m$ e $L_2 = 1\ m$.
Materiais: (a) betão $\lambda = 2.7\ W/(m.K)$; (b) bloco de betão; (c) reboco (2 cm) $\lambda = 0.43\ W/(m.K)$; (d) poliestireno expandido moldado (6 cm) $\lambda = 0.033\ W/(m.K)$.
Resistência térmica superficial exterior $R''_{se} = 0.04\ m^2K/W$, resistência térmica superficial interior $R''_{si} = 0.13\ m^2K/W$.

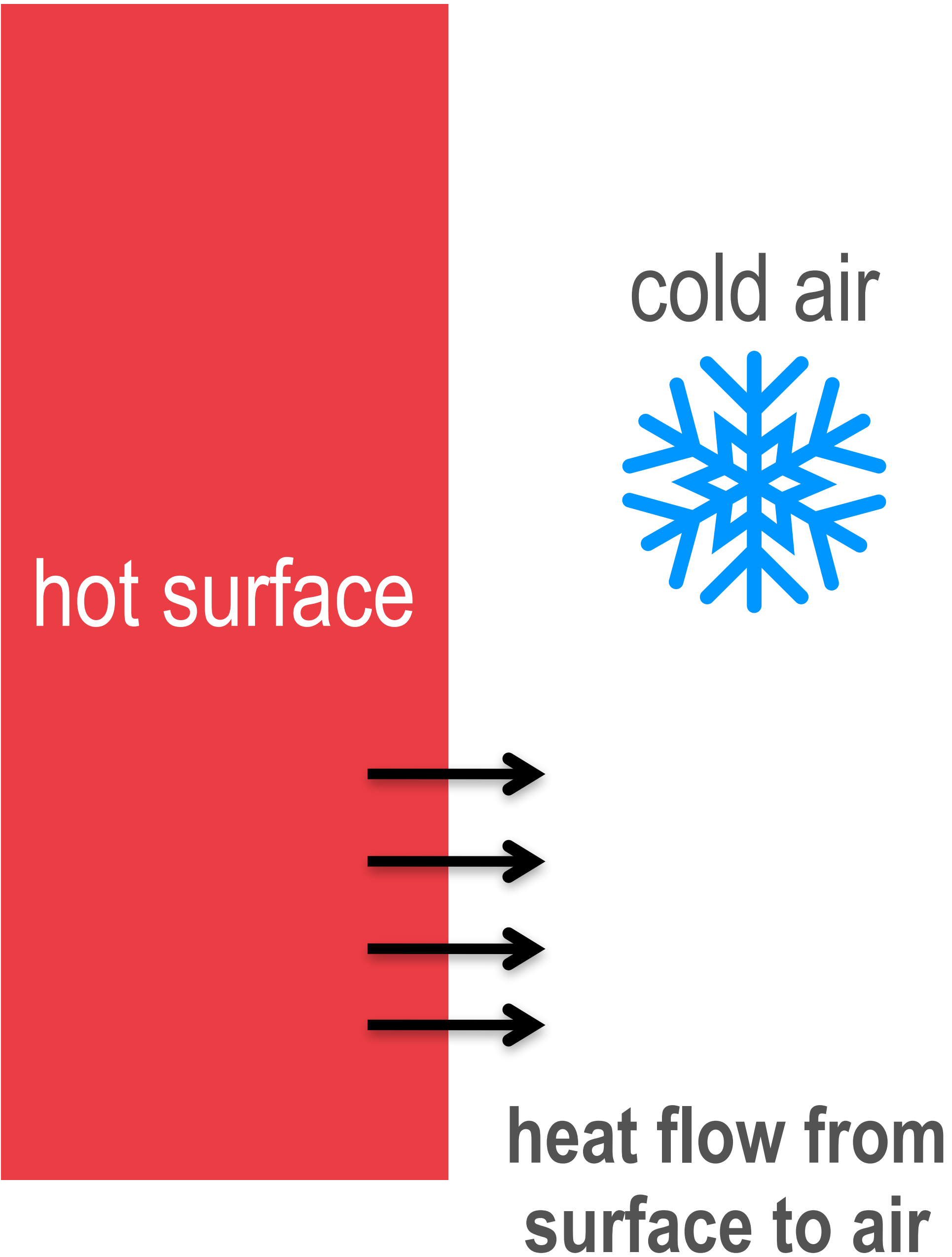
a) Calcular o coeficiente de transmissão térmica (U) da parede em A, considerando que os blocos de betão (elemento b) possuem 22 cm de espessura e uma resistência térmica unitária de 0.30 m^2K/W.

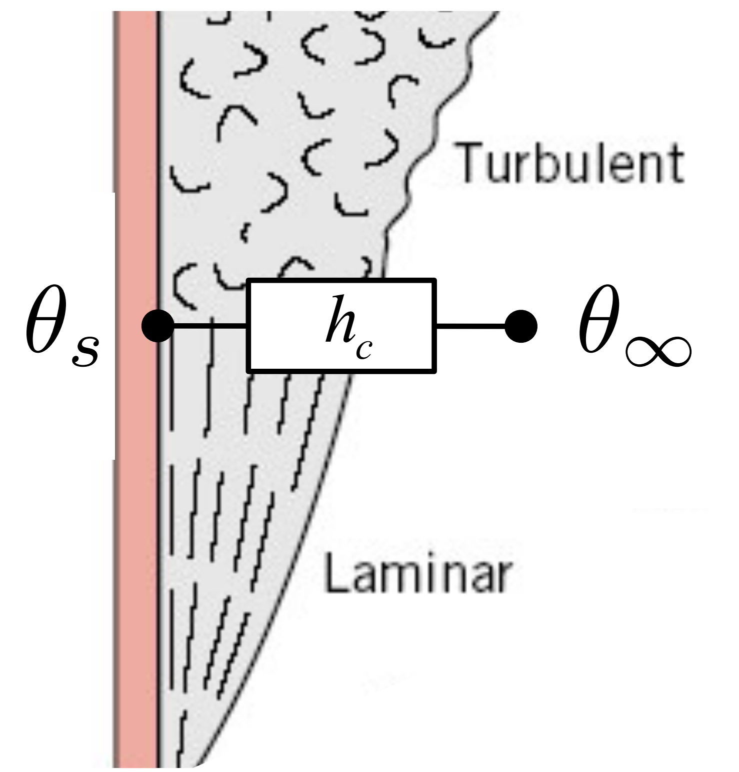
b) Calcular o coeficiente de transmissão térmica linear (Ψ) da junção entre as duas paredes, sabendo que, pelo método de diferenças finitas, em condições de regime permanente e para uma diferença de temperatura entre o interior e o exterior de 10°C, foi obtida uma taxa de calor de 30 W que atravessa o conjunto das superfícies 2 e 3 em A.

c) O que se pode esperar do valor de Ψ, caso o isolamento térmico tivesse sido colocado entre dois panos de blocos de betão com 11 cm de espessura e resistência térmica de 0.15 m^2K/W, conforme o esquema B? Justificar.


5
4
3
2
1
6
A
5
4
3
2
1
6
B
Temp [°C]
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0

Surface-to-air heat transfer





Ciências ULisboa

$$q_c = h_c A(\theta_s - \theta_\infty)$$

Convective heat transfer coefficient [W/(m²K)]

Heat flow rate [W]

Quantification of convective heat transfer

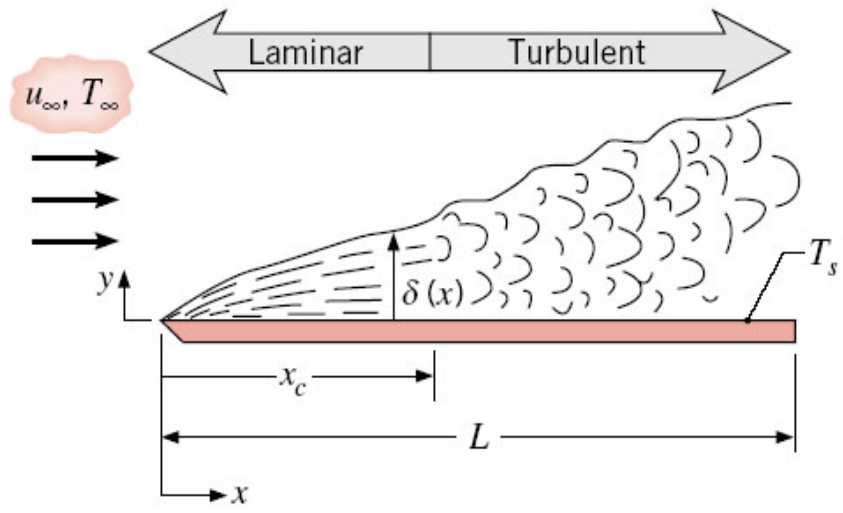
Empirical correlation

Complex problems

h_c depends on:

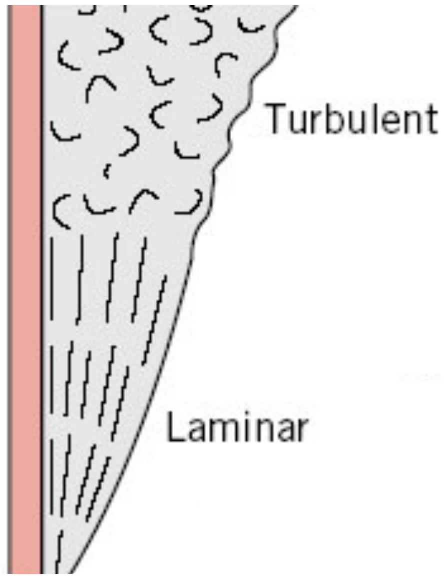
- fluid properties
- geometry
- velocity
- turbulent/laminar flow
- forced/natural convection
- temperature

Forced convection



$$h_c = Nu\frac{\lambda}{L}$$

Free convection



$$Nu = f(\mathrm{Re}, \mathrm{Pr})$$

Re Reynolds number: dimensionless parameter which relates inertia force and viscous friction

$$Nu = f(Gr, \mathrm{Pr})$$

Gr Grasshof number replaces the Reynolds for free convection, where inertia force depends on temperature difference

$$\mathrm{Nu} = \frac{h_c L}{\lambda}$$ Nusselt number

$$\mathrm{Pr} = \frac{c\mu}{\lambda}$$ Prandtl number

$$\mathrm{Re} = \frac{\rho U_{\infty L}}{\mu}$$ Reynolds number

$$\mathrm{Gr} = \frac{g\beta\Delta\theta\rho^2 L^3}{\mu^2}$$ Grashof number

$$\mathrm{Ra} = \mathrm{Gr} \cdot \mathrm{Pr}$$ Rayleigh number

Fluid properties

- λ Thermal conductivity [W/(m.K)]
- c Specific heat [J/(kg.K)]
- ρ Density [kg/m^3]
- μ Viscosity [kg/(m.s)]
- β Volumetric thermal expansion coefficient [K^{-1}]

Local parameters

- L Characteristic length [m]
- U_∞ Fluid velocity [m/s]
- $\Delta\theta$ Surface-fluid temperature difference [K]

Ciências ULisboa

Vertical surface

Laminar

$$\mathrm{Ra} < 10^9$$

$$\mathrm{Nu} = 0.59\mathrm{Ra}^{1/4}$$

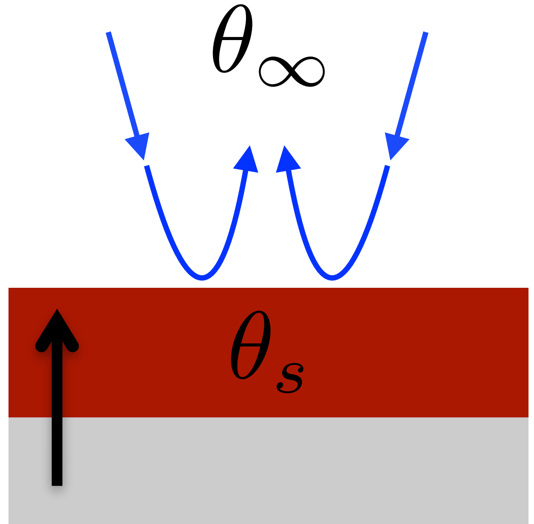
Turbulent

$$\mathrm{Ra} > 10^9$$

$$\mathrm{Nu} = 0.10\mathrm{Ra}^{1/3}$$

H is the characteristic length (L=H)
Air properties are evaluated at average temperature (surface and undisturbed air)

Heat flow upwards

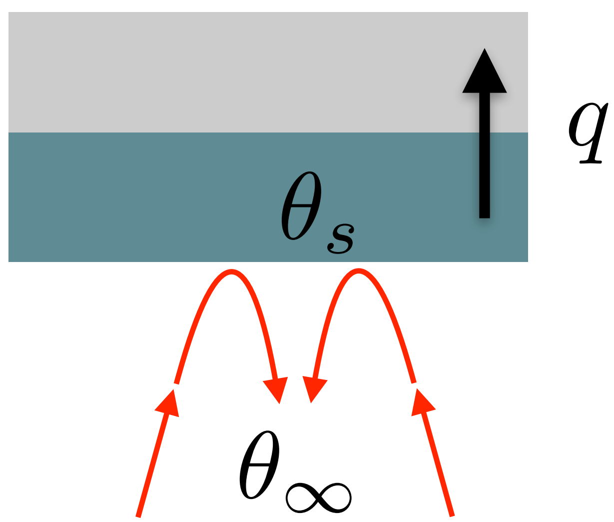


q Heated floor

$\theta_s > \theta_\infty$

Cooled ceiling

$\theta_s < \theta_\infty$



Laminar

$\mathrm{Ra} < 10^7$

$\mathrm{Nu} = 0.54\mathrm{Ra}^{1/4}$

Turbulent

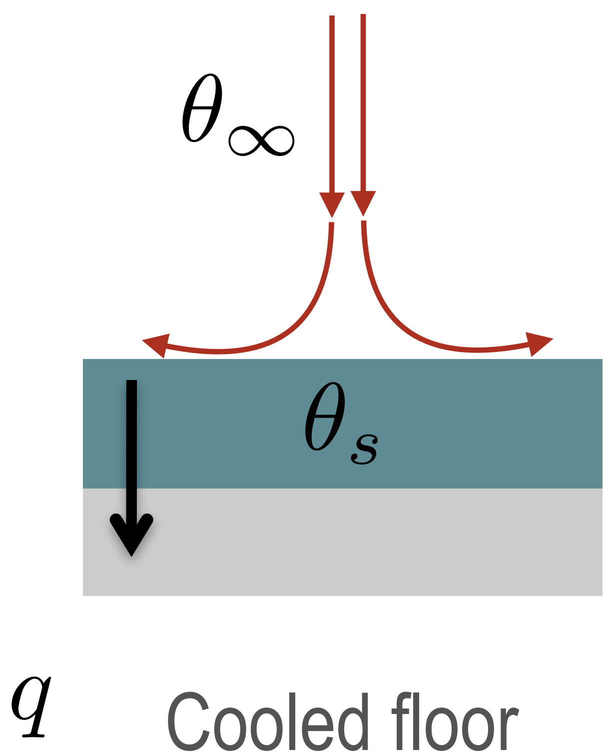
$\mathrm{Ra} > 10^7$

$\mathrm{Nu} = 0.15\mathrm{Ra}^{1/3}$

$\dfrac{A_s}{P}$ is the characteristic length

Air properties are evaluated at average temperature (surface and undisturbed air)

Heat flow downwards

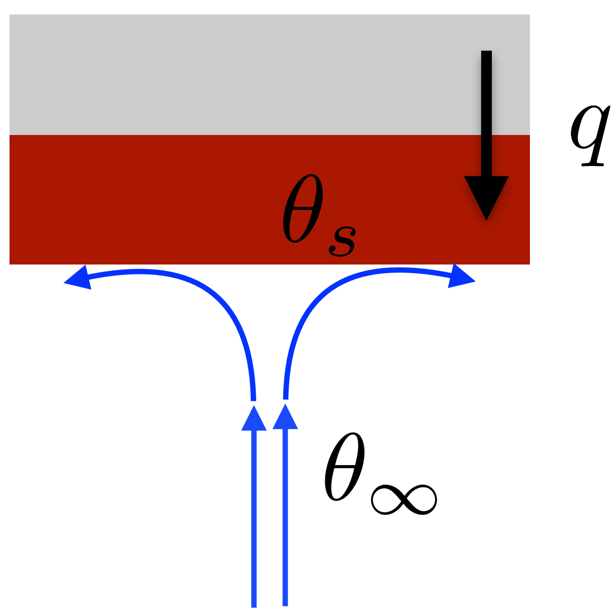


Cooled floor

$$\theta_s < \theta_\infty$$

Heated ceiling

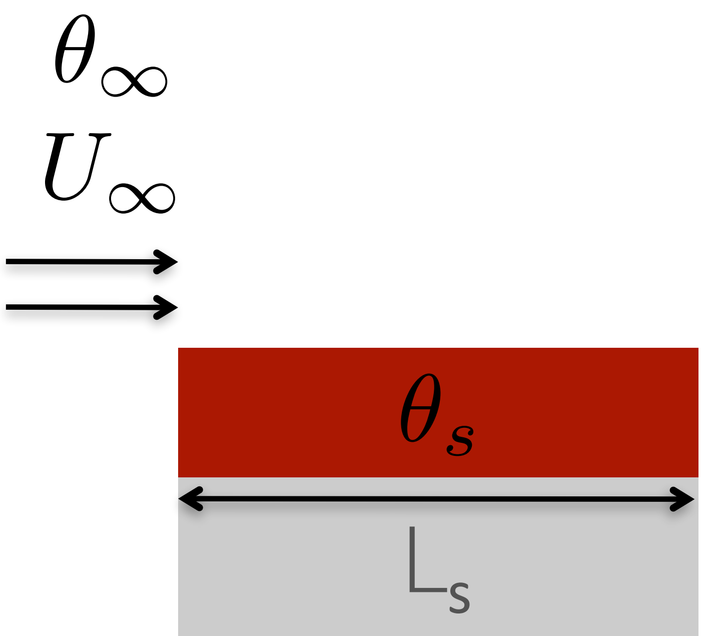
$$\theta_s > \theta_\infty$$



$$\mathrm{Nu} = 0.27\mathrm{Ra}^{1/4}$$

$\dfrac{A_s}{P}$ is the characteristic length

Air properties are evaluated at average temperature (surface and undisturbed air)



Laminar

$$\mathrm{Re} < 5 \times 10^5$$

$$\mathrm{Nu} = 0.664\mathrm{Re}^{1/2}\mathrm{Pr}^{1/3}$$ (average value)

Laminar and turbulent

$$\mathrm{Re} > 5 \times 10^5$$

$$\mathrm{Nu} = (0.037\mathrm{Re}^{4/5} - A)\mathrm{Pr}^{1/3}$$ (average value)

L_s is the characteristic length ($L=L_s$)

$$A \simeq 871$$

Surface	Heat flow	h_c
External		20
Internal	Horizontal (wall)	2.5
	Vertical upwards	5.0
	Vertical downwards	0.7

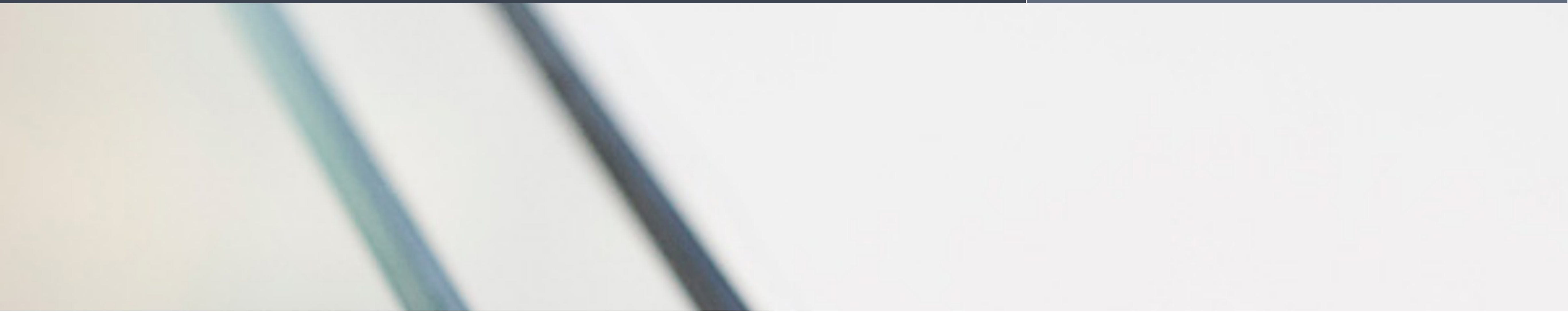
External surface

$h_c = 4 + 4U_\infty$ $U_\infty \leq 5$ [m/s]

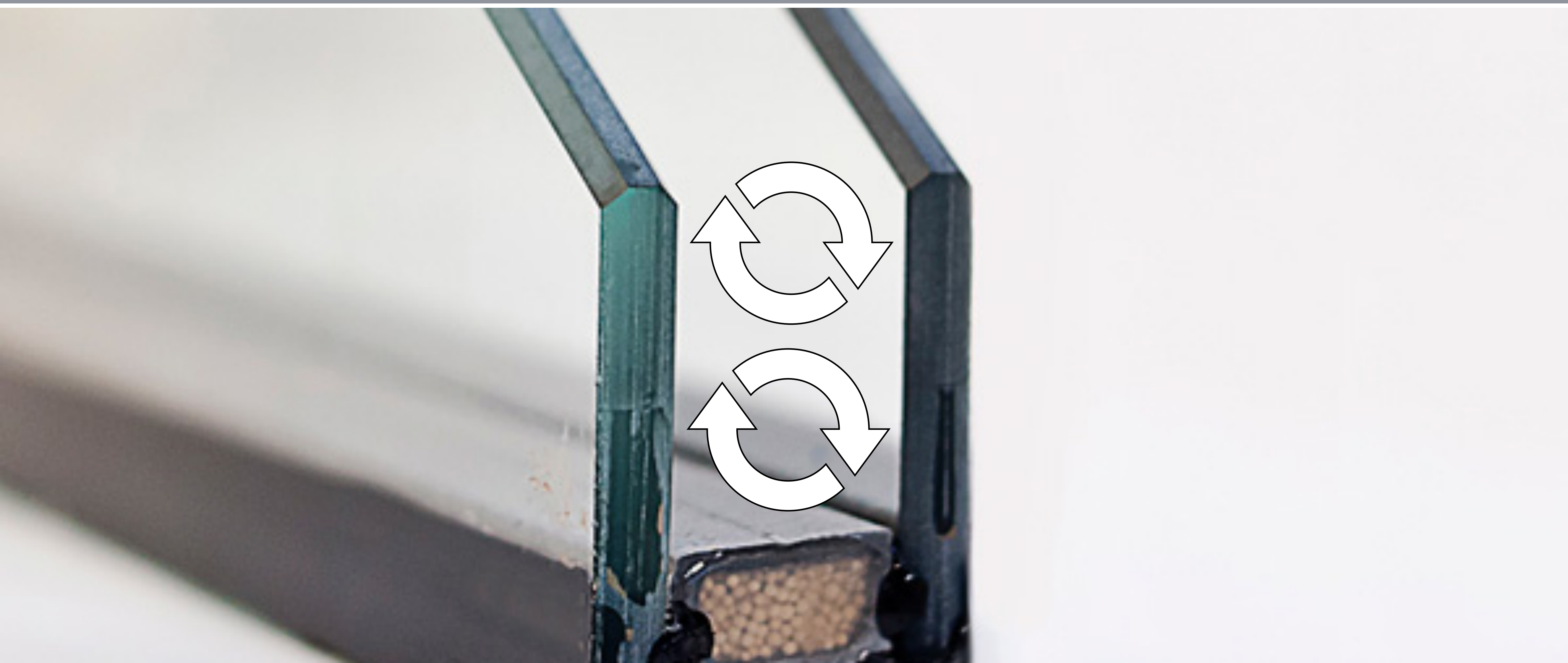
Surface	Heat flow	h_c	h_s	R_s
External		20	25	0.04
Internal	Horizontal (wall)	2.5	7.5	0.13
	Vertical upwards	5	10	0.10
	Vertical downwards	0.7	5.7	0.17

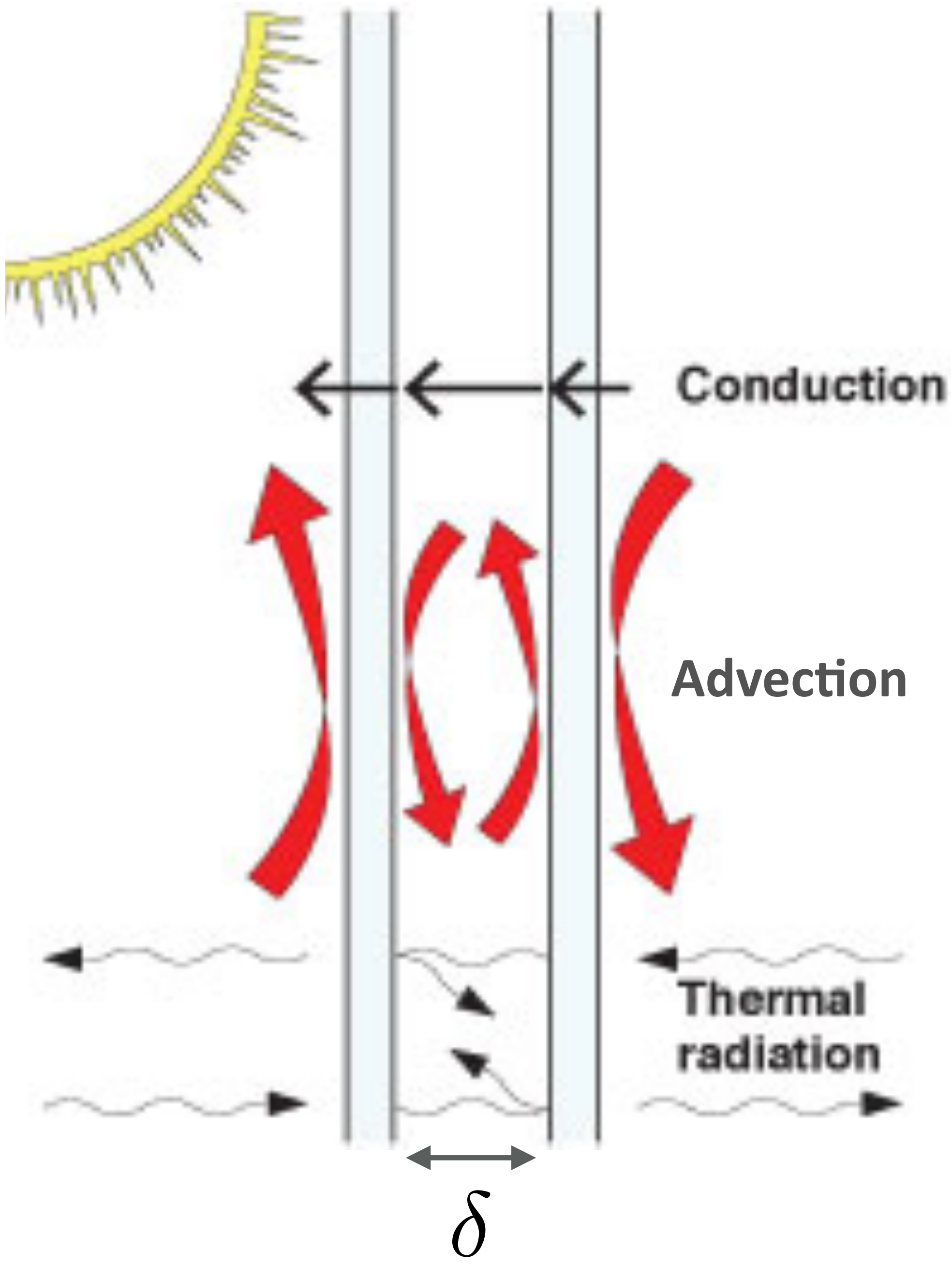
$$h_s = h_r + h_c$$

$$R_s = 1/h_s$$



Surface-to-surface in air cavities





Ciências ULisboa

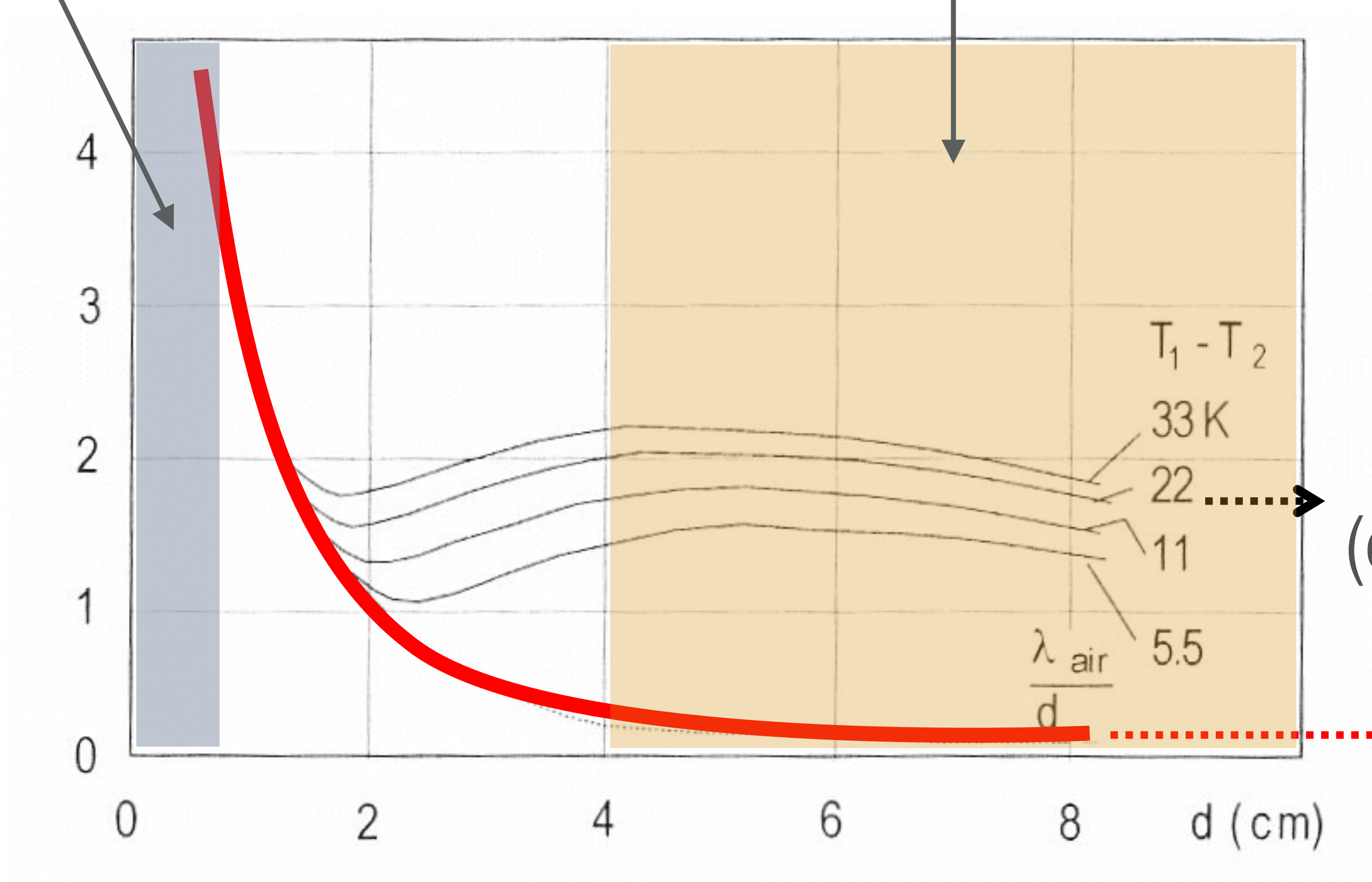
negligible advection

$\mathrm{Nu} \simeq 1$

negligible diffusion

h_c

[W/(m².K)]



heat convection (diffusion+advection)

only heat conduction $\dfrac{\lambda_a}{\delta}$

* radiative effects not included

$$H/\delta > 10$$

$$H/\delta < 40$$

δ is the characteristic length

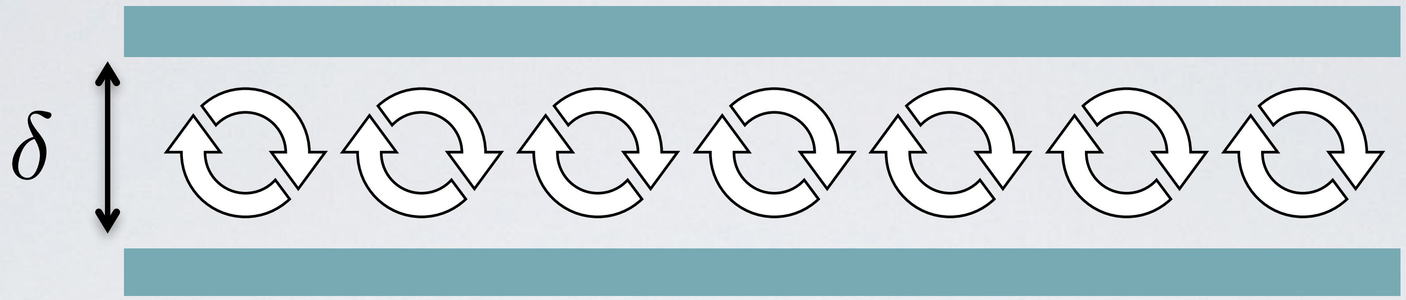
$$2 \times 10^3 \lesssim \mathrm{Ra} \lesssim 2 \times 10^5$$

$$\mathrm{Nu}_\delta = 0.197\mathrm{Ra}^{1/4} \left(\frac{H}{\delta}\right)^{-1/9}$$

$$2 \times 10^5 \lesssim \mathrm{Ra} \lesssim 10^7$$

$$\mathrm{Nu}_\delta = 0.073\mathrm{Ra}^{1/3} \left(\frac{H}{\delta}\right)^{-1/9}$$

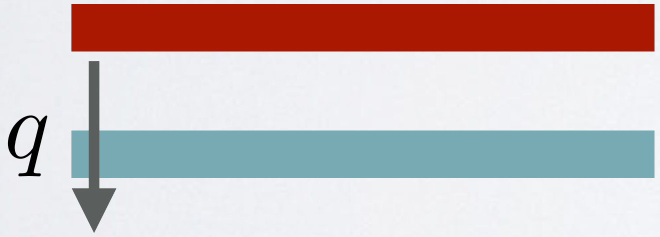
Ciências ULisboa



δ is the characteristic length

heat flow downwards

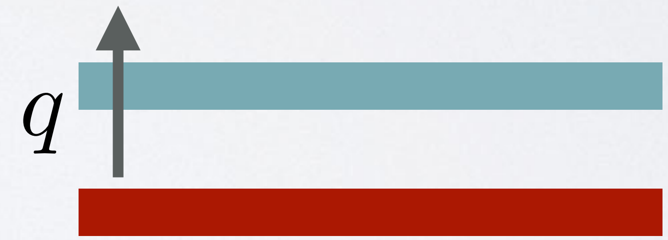
$$\mathrm{Nu} = 1$$



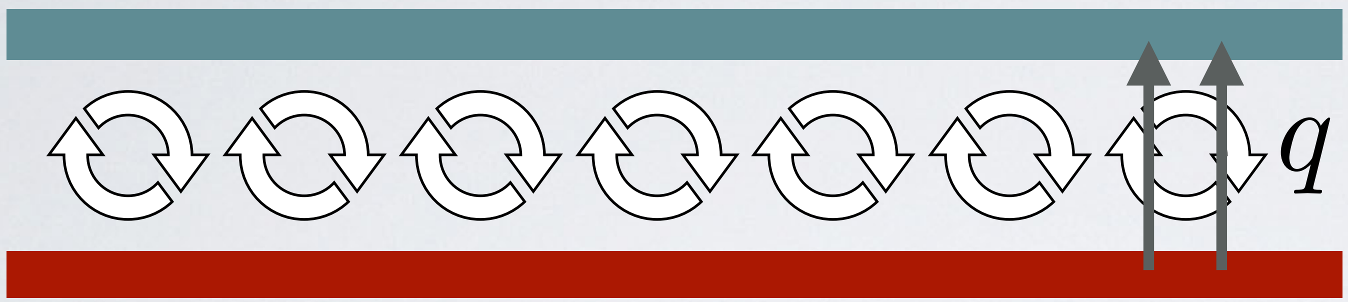
heat flow upwards

$$1.7 \times 10^3 \lesssim \mathrm{Ra} \lesssim 7 \times 10^3$$

$$\mathrm{Nu}_\delta = 0.059\mathrm{Ra}^{2/5}$$

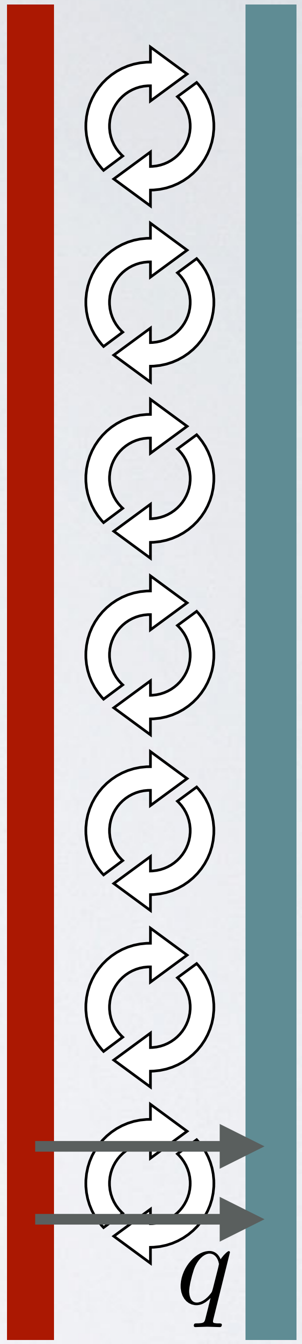


upward heat flow



downward heat flow

vertical air cavity

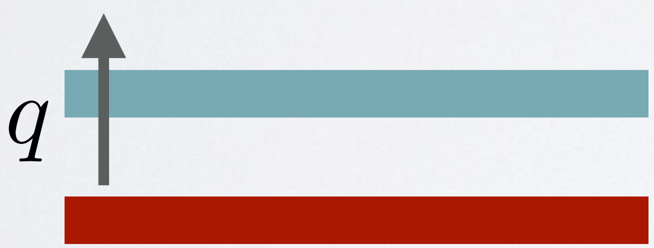


δ is the characteristic length

heat flow upwards

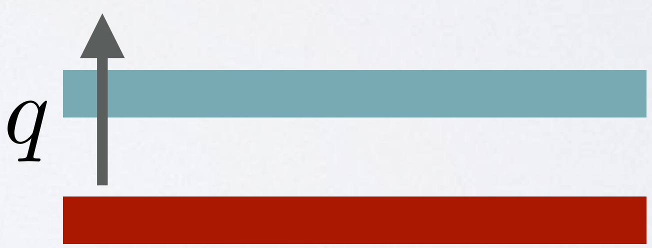
$$7 \times 10^3 \lesssim \mathrm{Ra} \lesssim 3.2 \times 10^5$$

$$\mathrm{Nu}_\delta = 0.212\mathrm{Ra}^{1/4}$$



$$\mathrm{Ra} \gtrsim 3.2 \times 10^5$$

$$\mathrm{Nu}_\delta = 0.061\mathrm{Ra}^{1/3}$$



surface-to-surface heat transfer* [W/(m²K)]

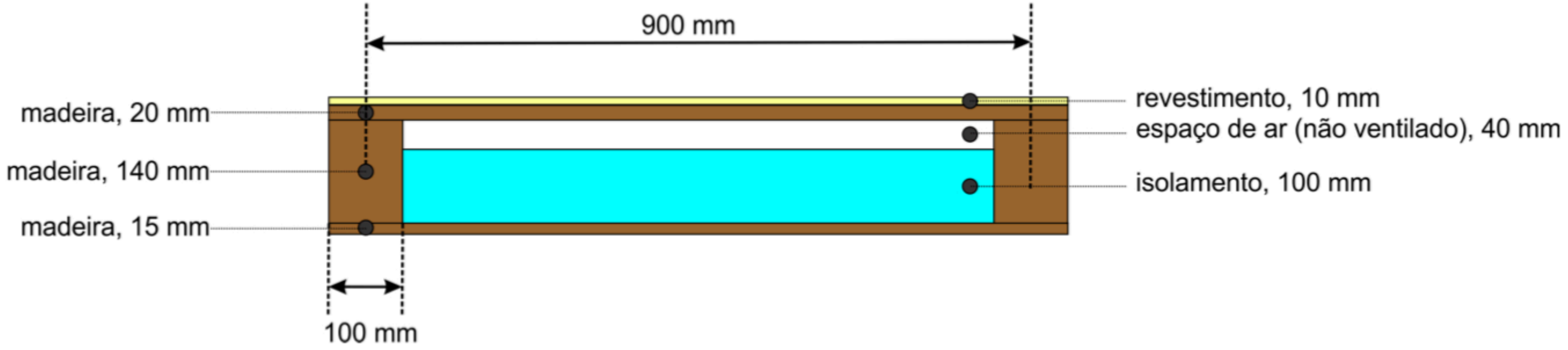
Surfaces	Heat flow	Gap width [mm]						
		5	10	15	25	50	100	300
Vertical		4.8	2.3	1.6	1.2	1.2	1.2	1.2
Horizontal	upwards	4.8	2.4	1.9	1.9	1.9	1.9	1.9
Horizontal	downwards	4.8	2.4	1.6	0.9	0.4	0.2	0

*only for air gaps, not valid for other gases

Exercício 2.2.3 Uma laje interior é composta pela sobreposição de vários materiais, sendo que o padrão unitário com dimensões $90 \times 100\ cm$ repete-se em toda a área da laje. Considerando condições de regime permanente, calcular o coeficiente de transmissão térmica (U) para a situação em que se verifica:

a) fluxo de calor ascendente

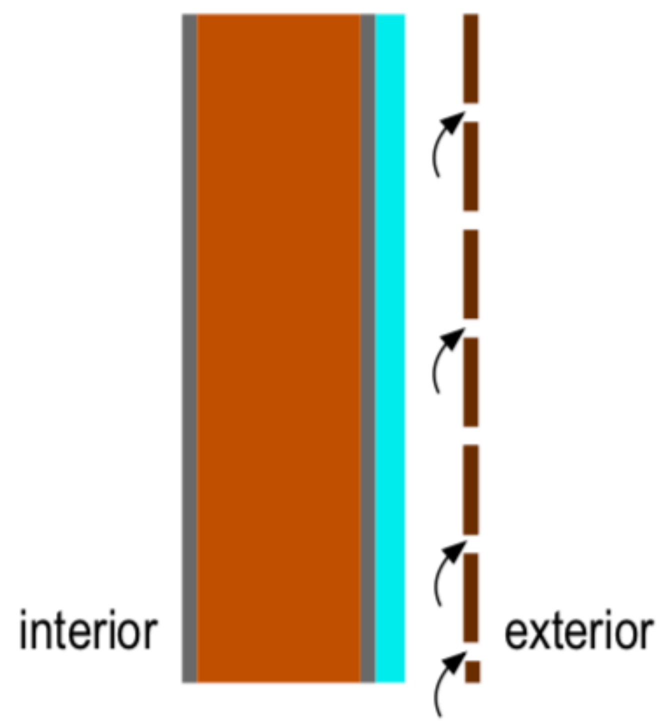
b) fluxo de calor descendente



Propriedades dos materiais:

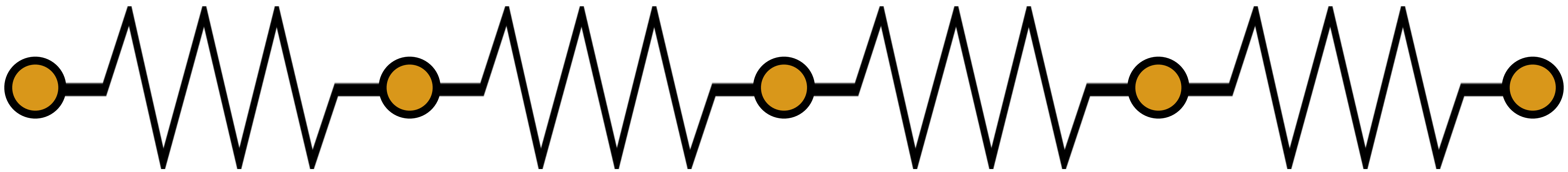
- Madeira: $\lambda = 0.14\ W/(mK)$
- Isolamento: $\lambda = 0.044\ W/(mK)$
- Revestimento: $\lambda = 0.17\ W/(mK)$

Exercício 2.3.2 **Calcular o coeficiente de transferência térmica superficial** (U) da parede exterior representada na figura em que a caixa de ar é fortemente ventilada com ar exterior.

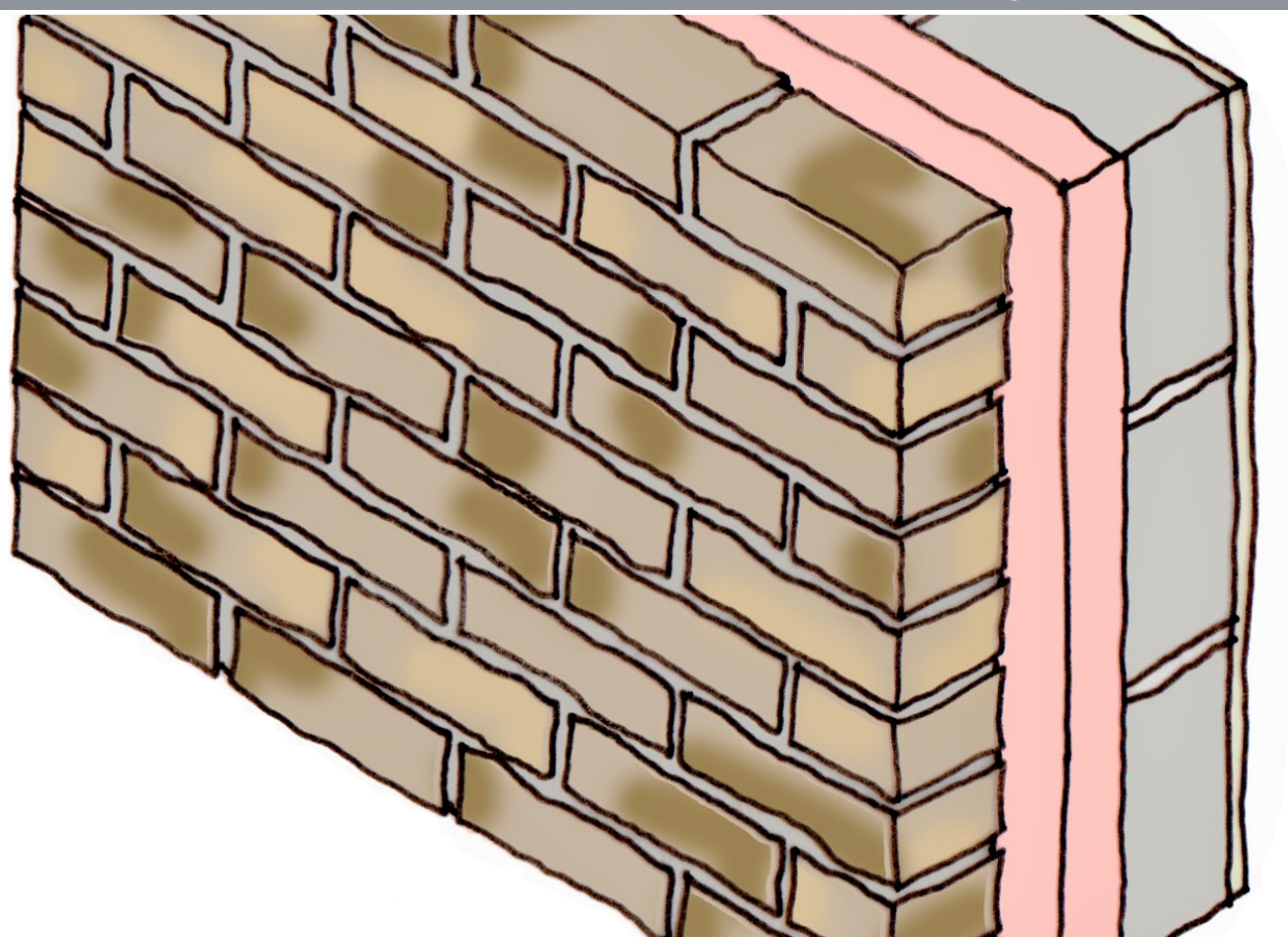


(do exterior para o interior)

1. madeira (2 cm),
2. cavidade ventilada,
3. lã mineral (4 cm), $\lambda = 0.04\ W/(mK)$,
4. reboco (2 cm), $\lambda = 1.15\ W/(mK)$,
5. tijolo cerâmico (22 cm), $R'' = 0.52\ m^2K/W$,
6. reboco (2 cm), $\lambda = 1.15\ W/(mK)$

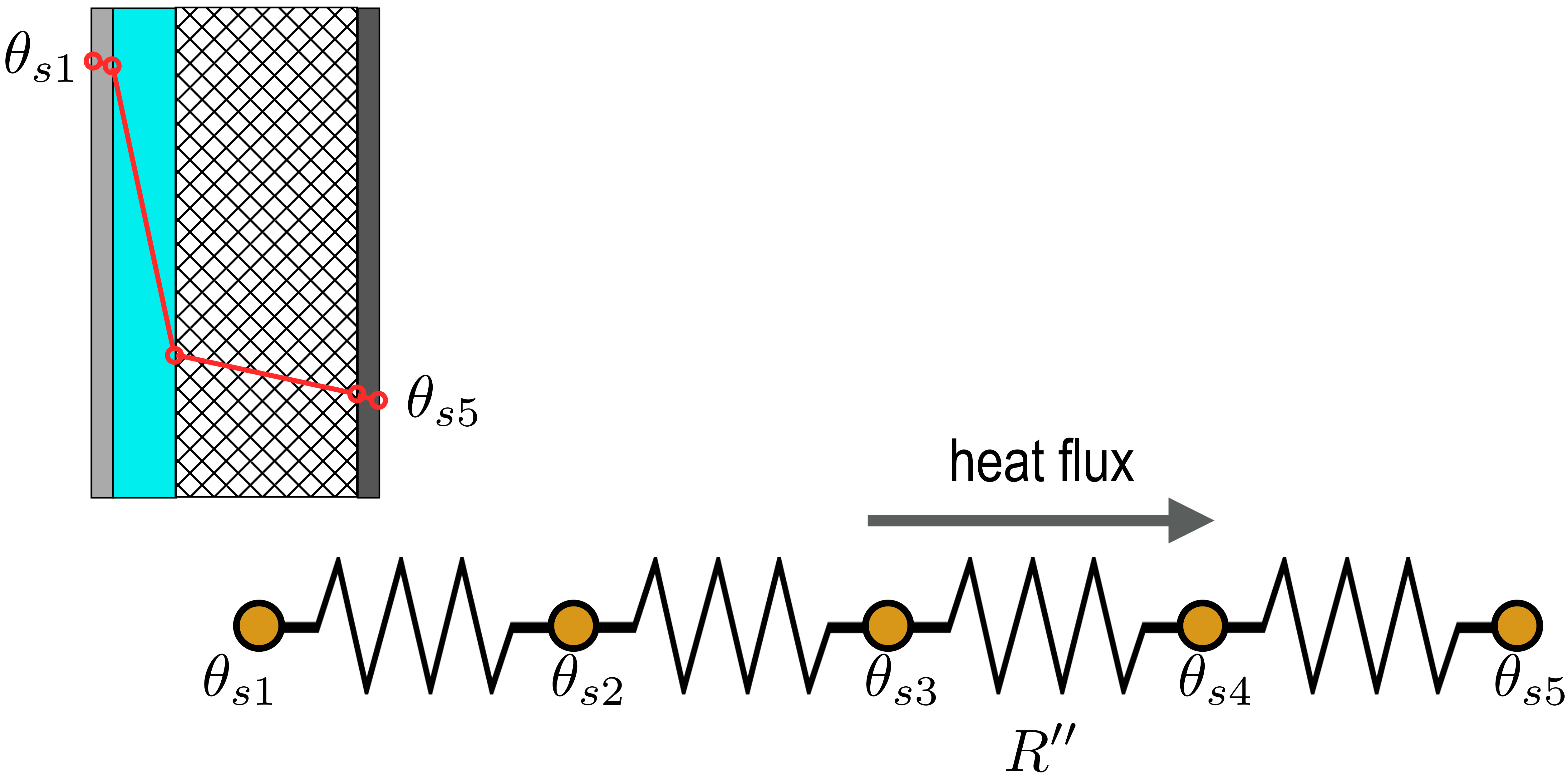


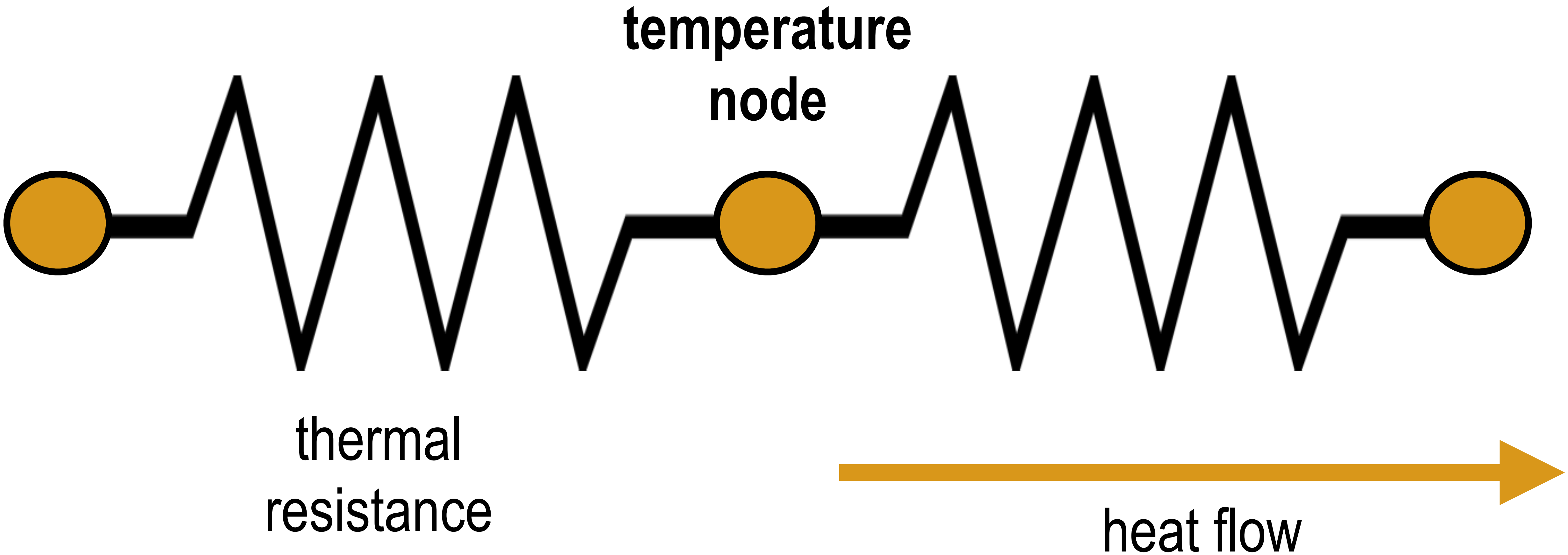
Thermal network analysis

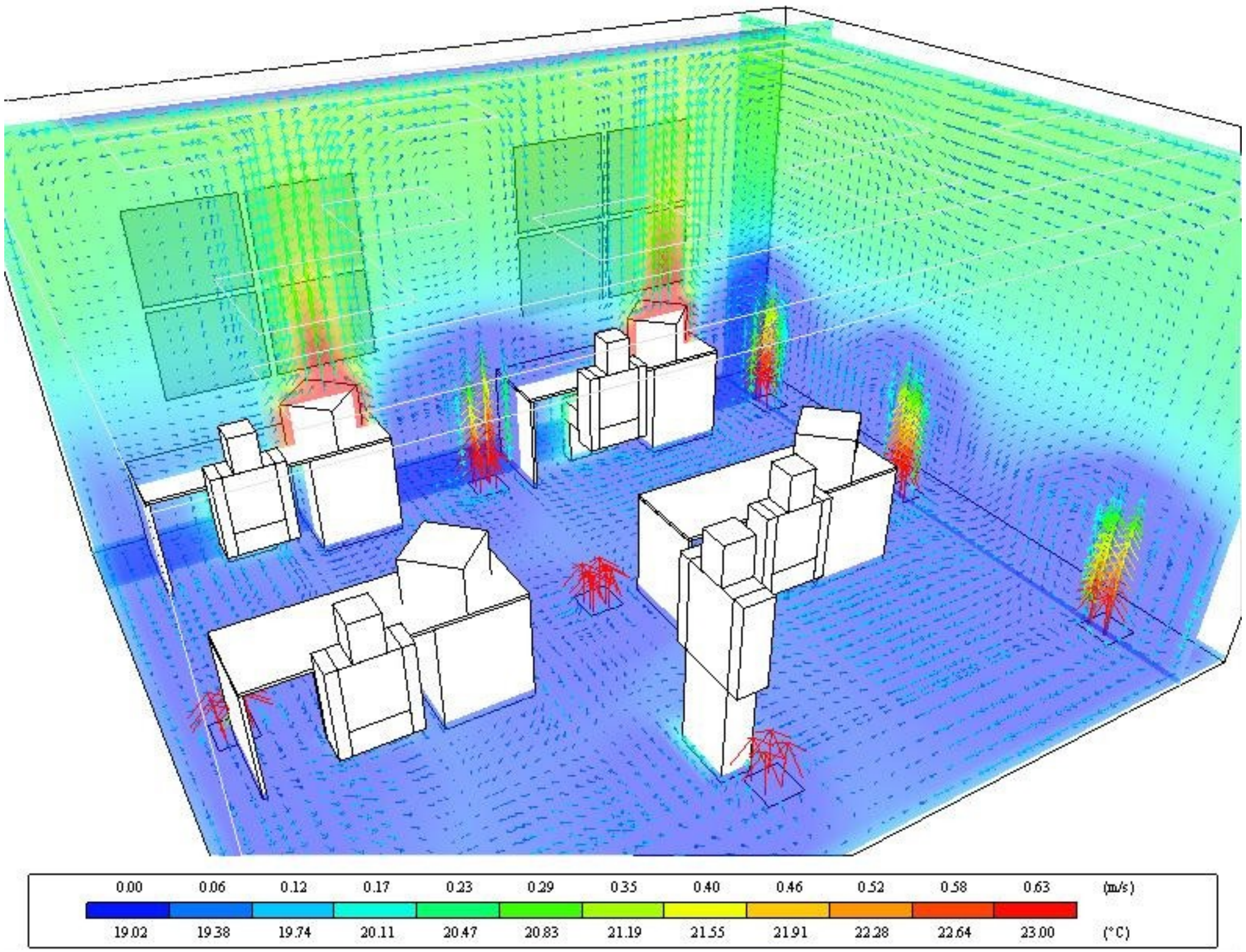


Ciências ULisboa

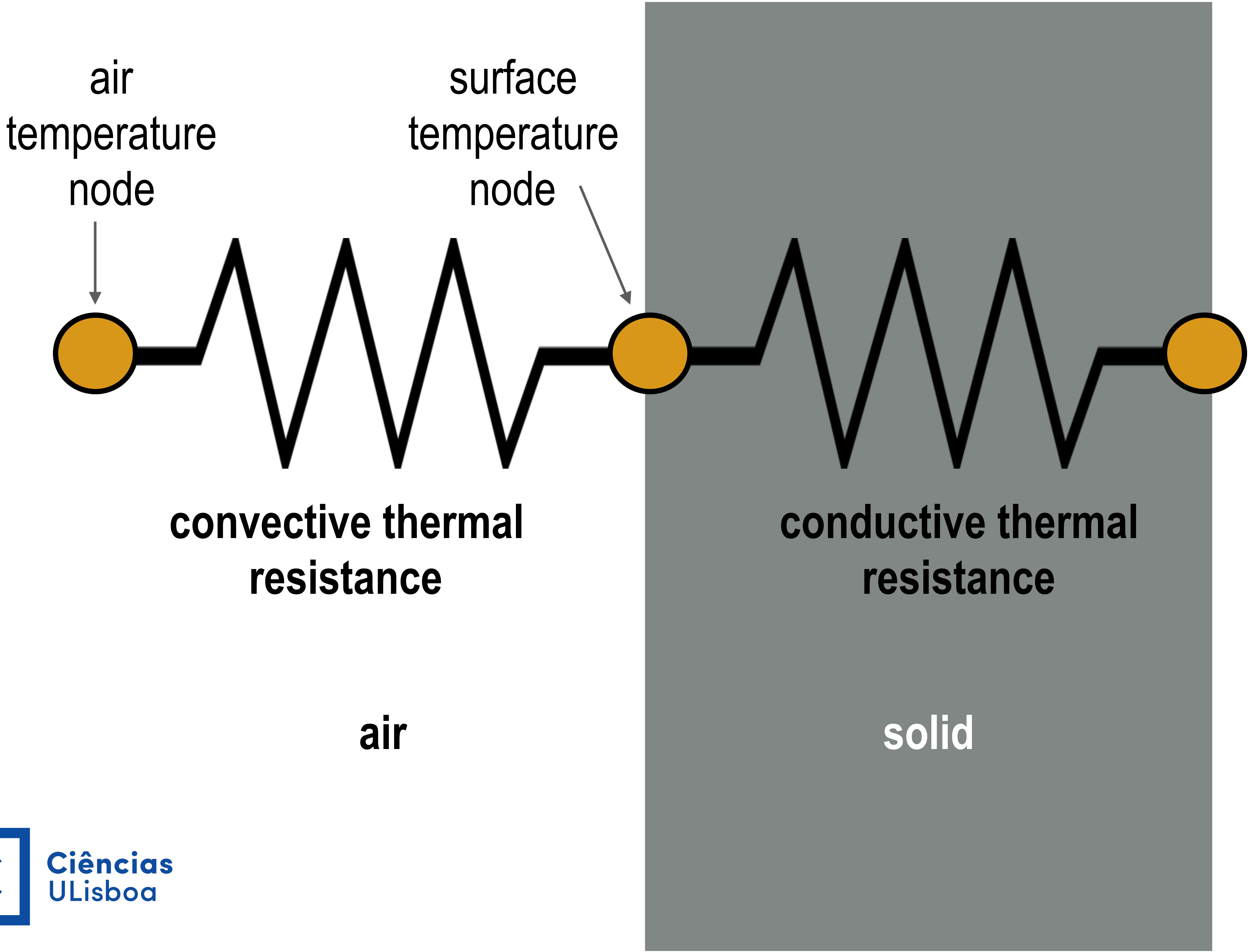
Network representation



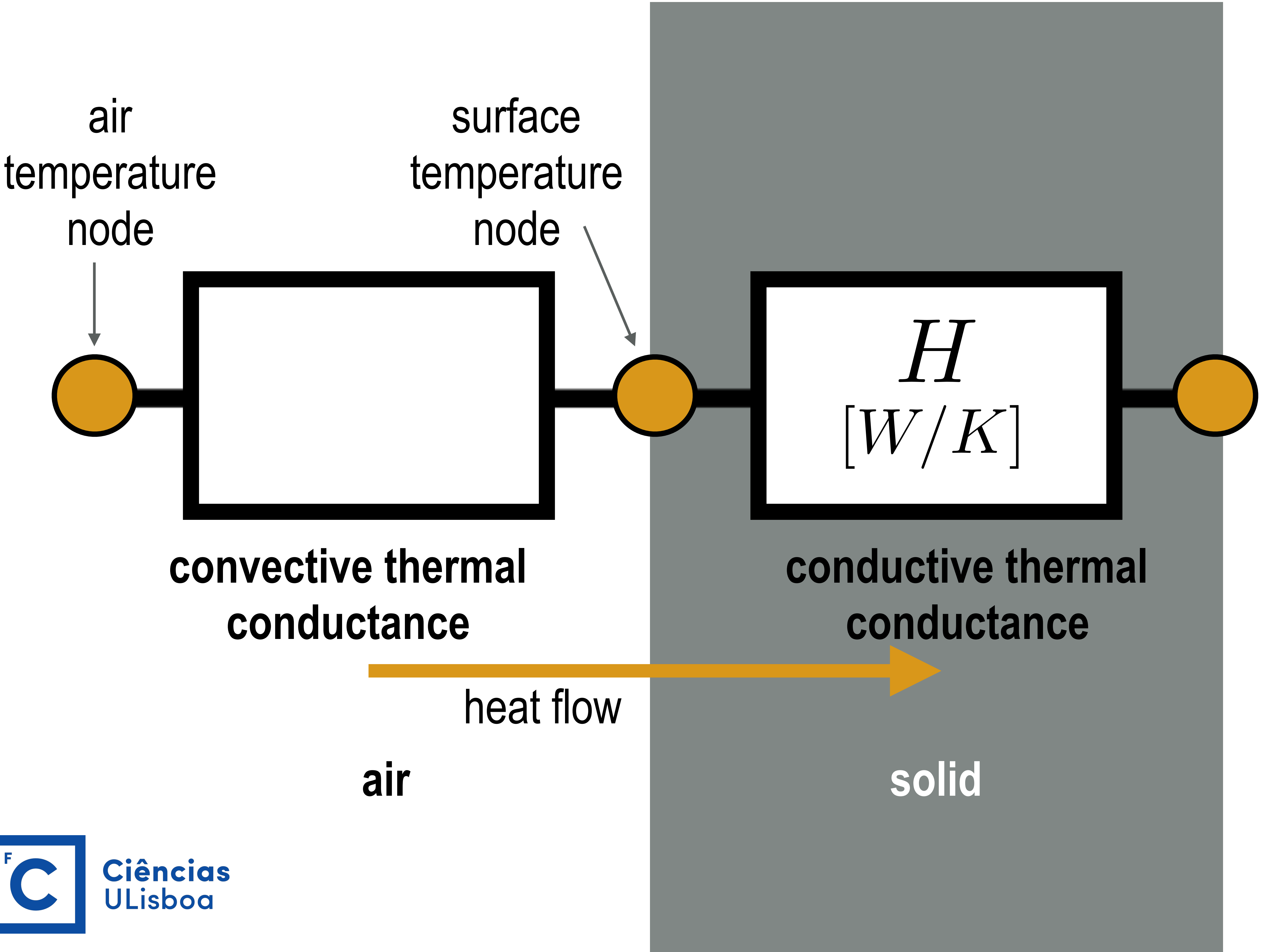

temperature
node
thermal
resistance
heat flow



room temperature node = spatial average temperature



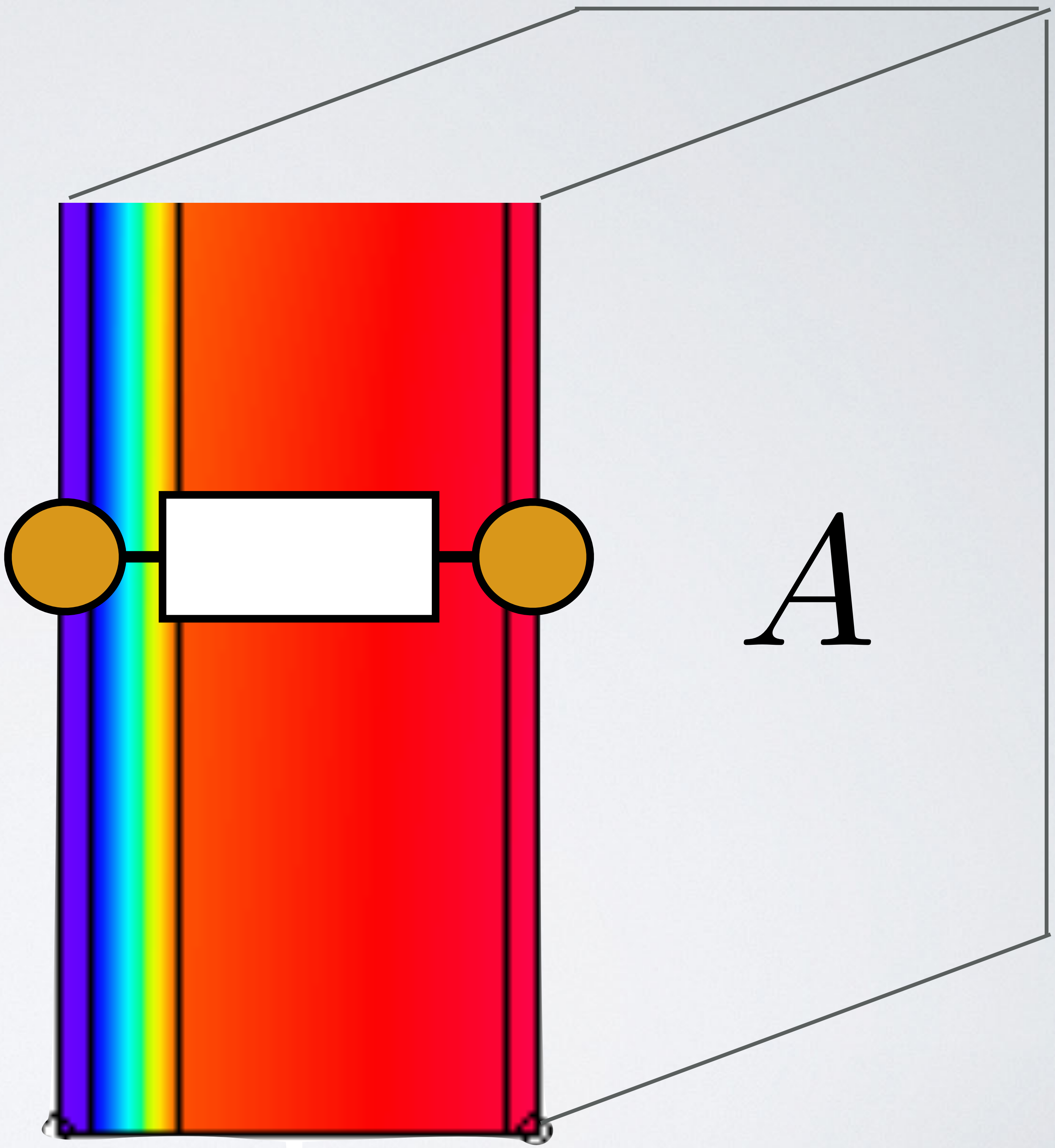
Ciências ULisboa

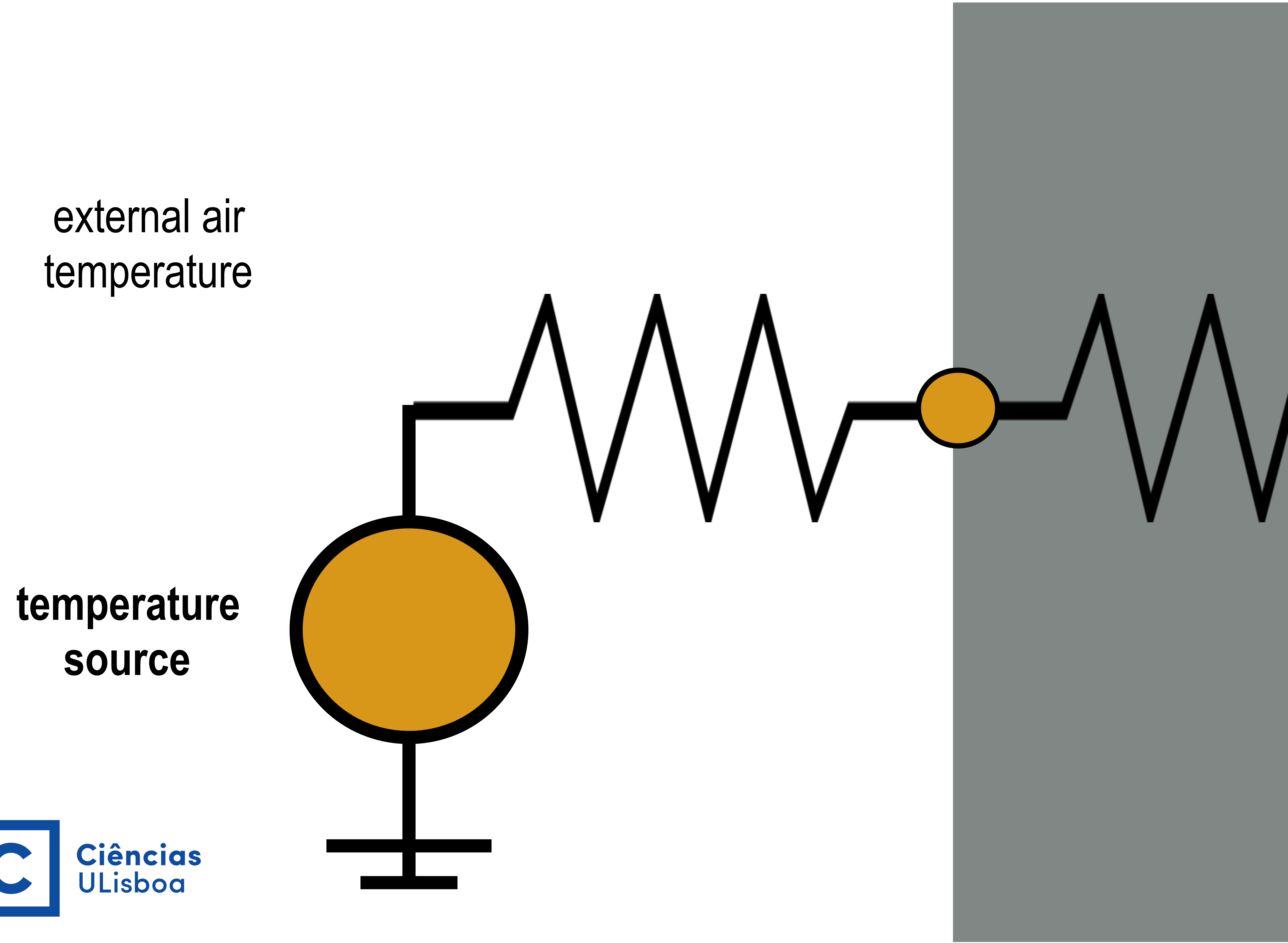

air temperature node
surface temperature node
H
$[W/K]$
convective thermal conductance
conductive thermal conductance
heat flow
air
solid

$$H = U\,A$$

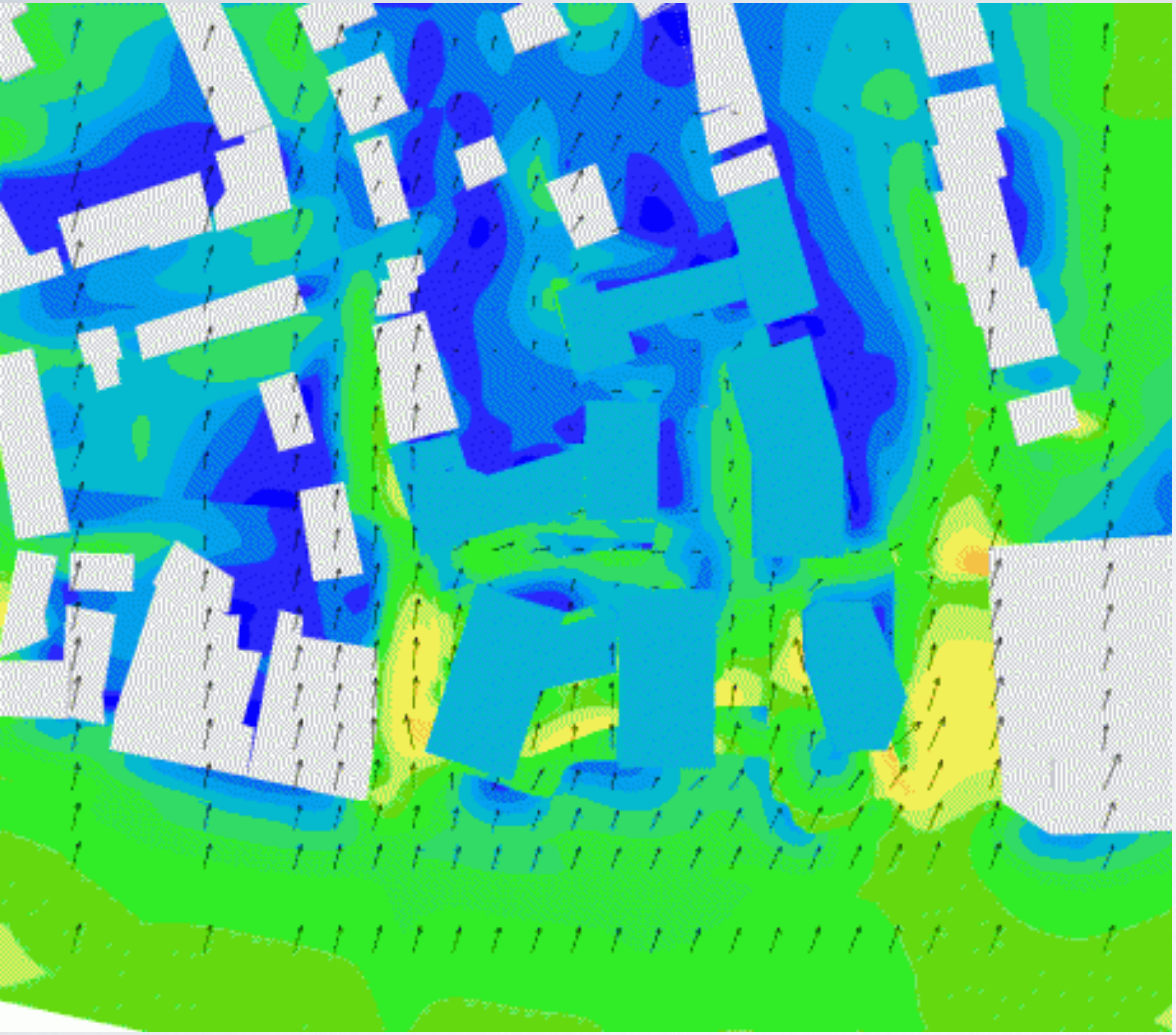
$$H = \frac{1}{R} = \frac{A}{R''}$$

$$[W/K]$$

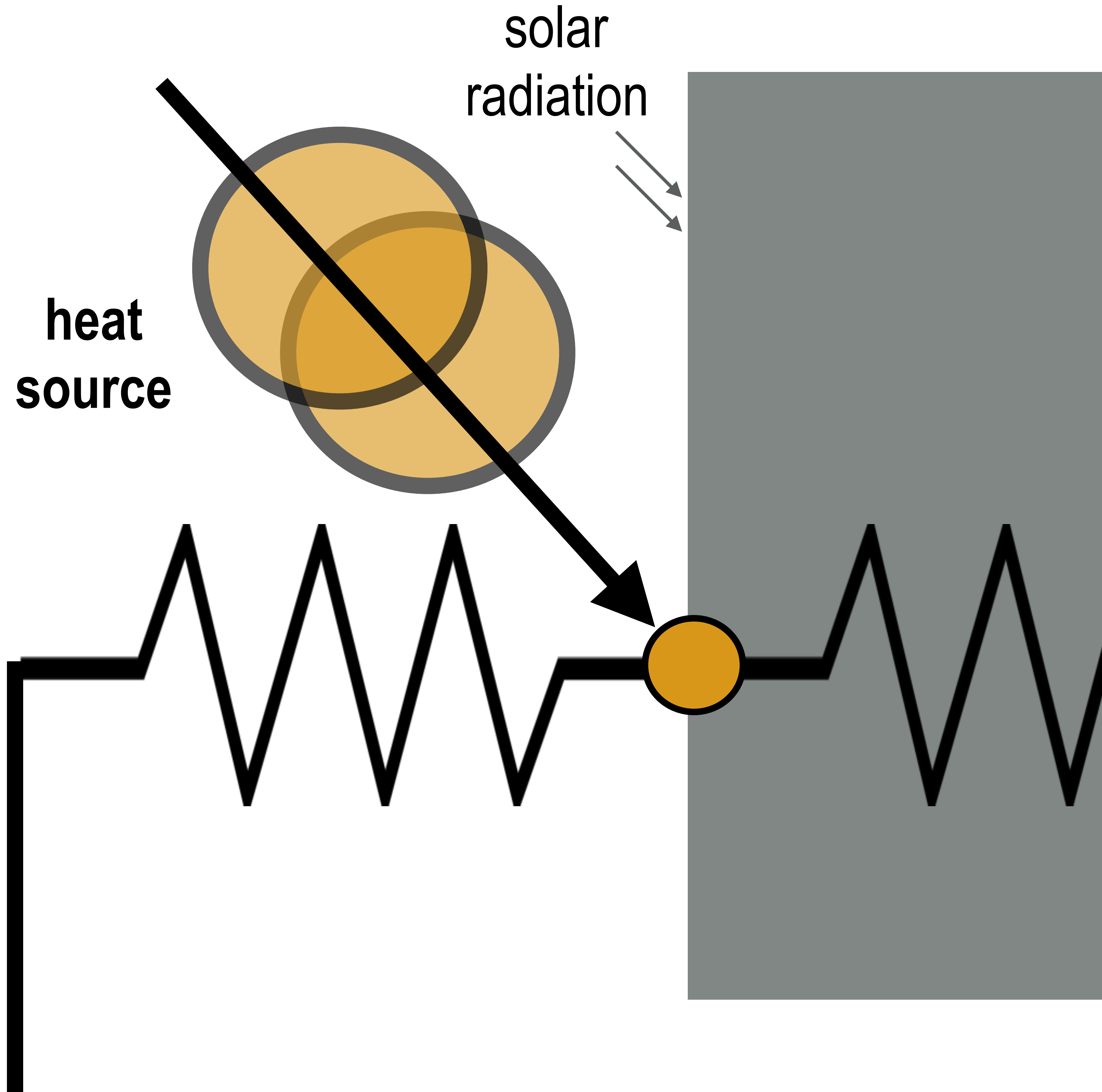




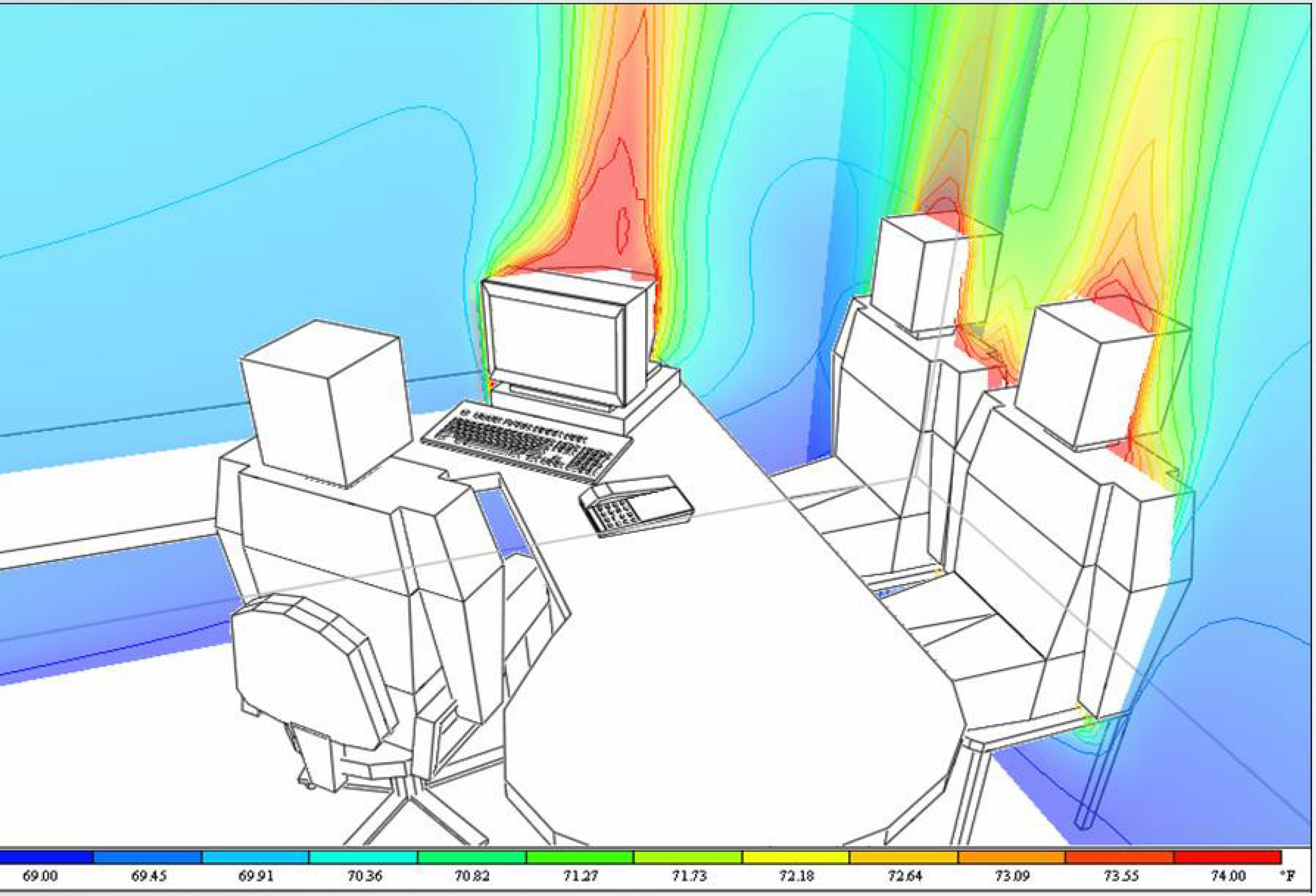
Ciências
ULisboa



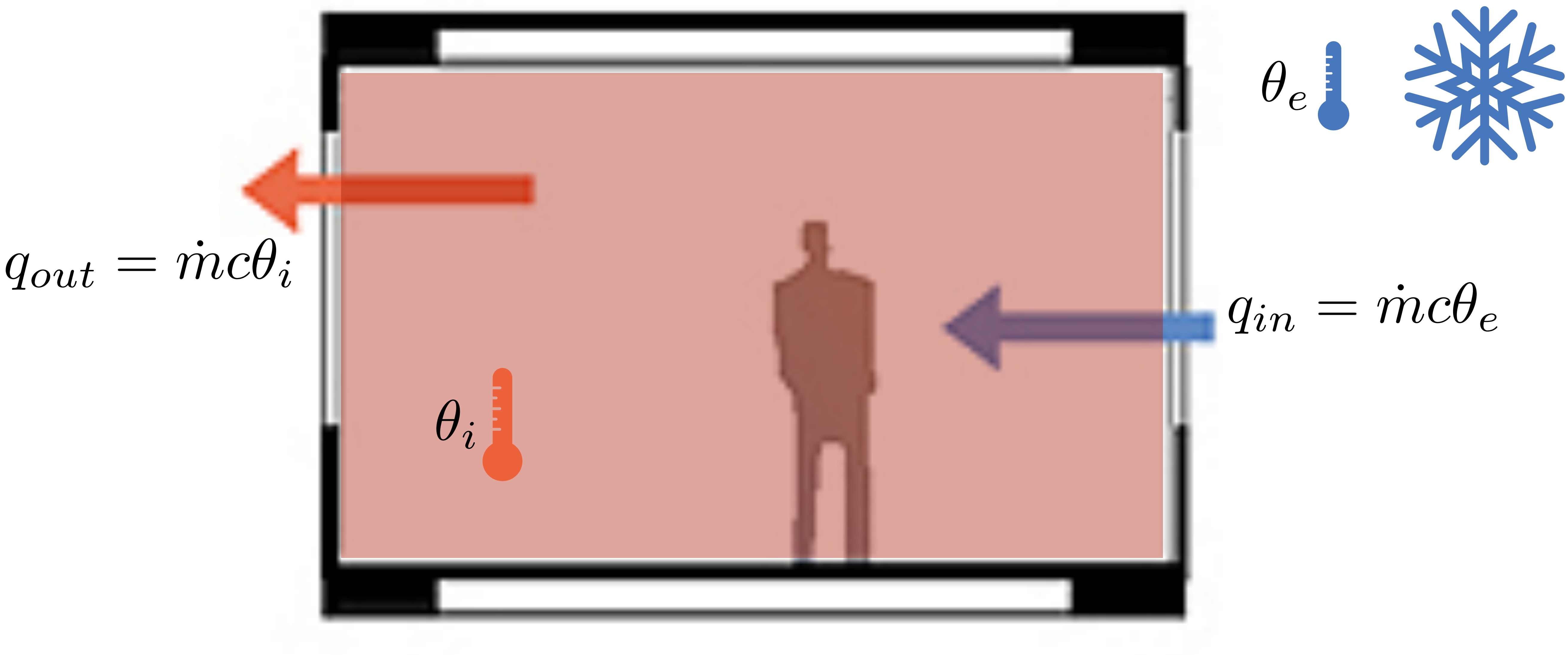
Despite local variations, **external air** can be considered a temperature source

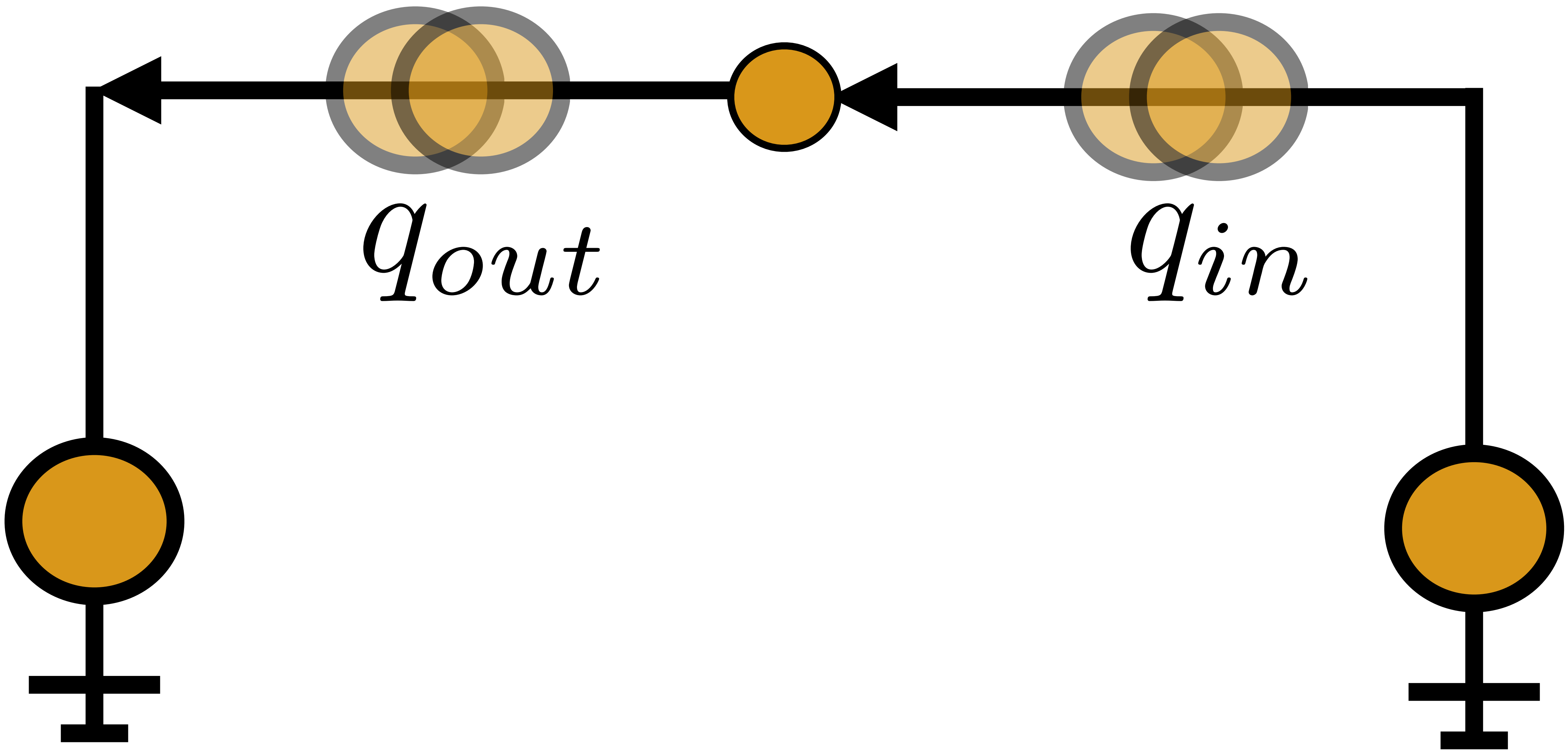


Ciências ULisboa

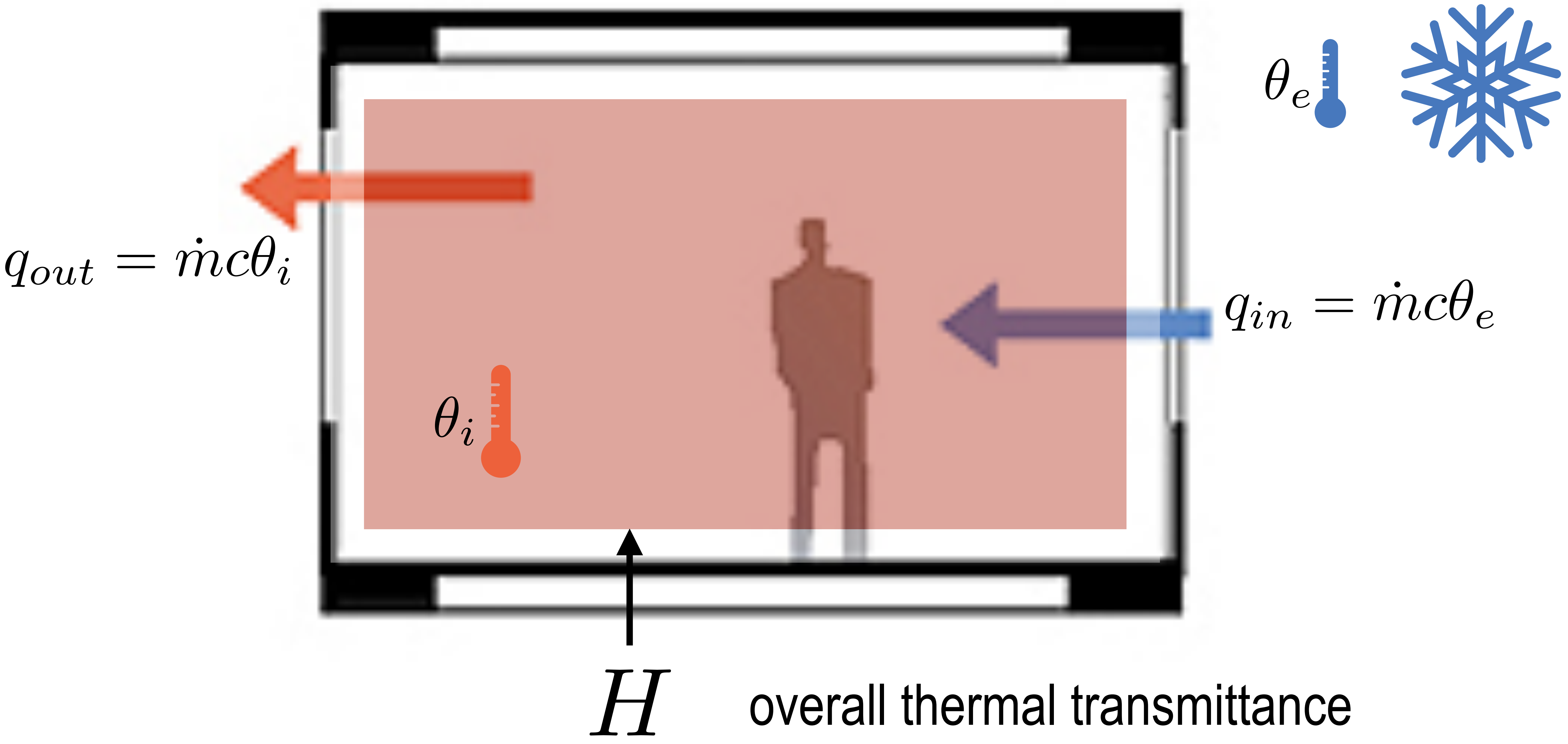


People and equipments are examples of heat sources

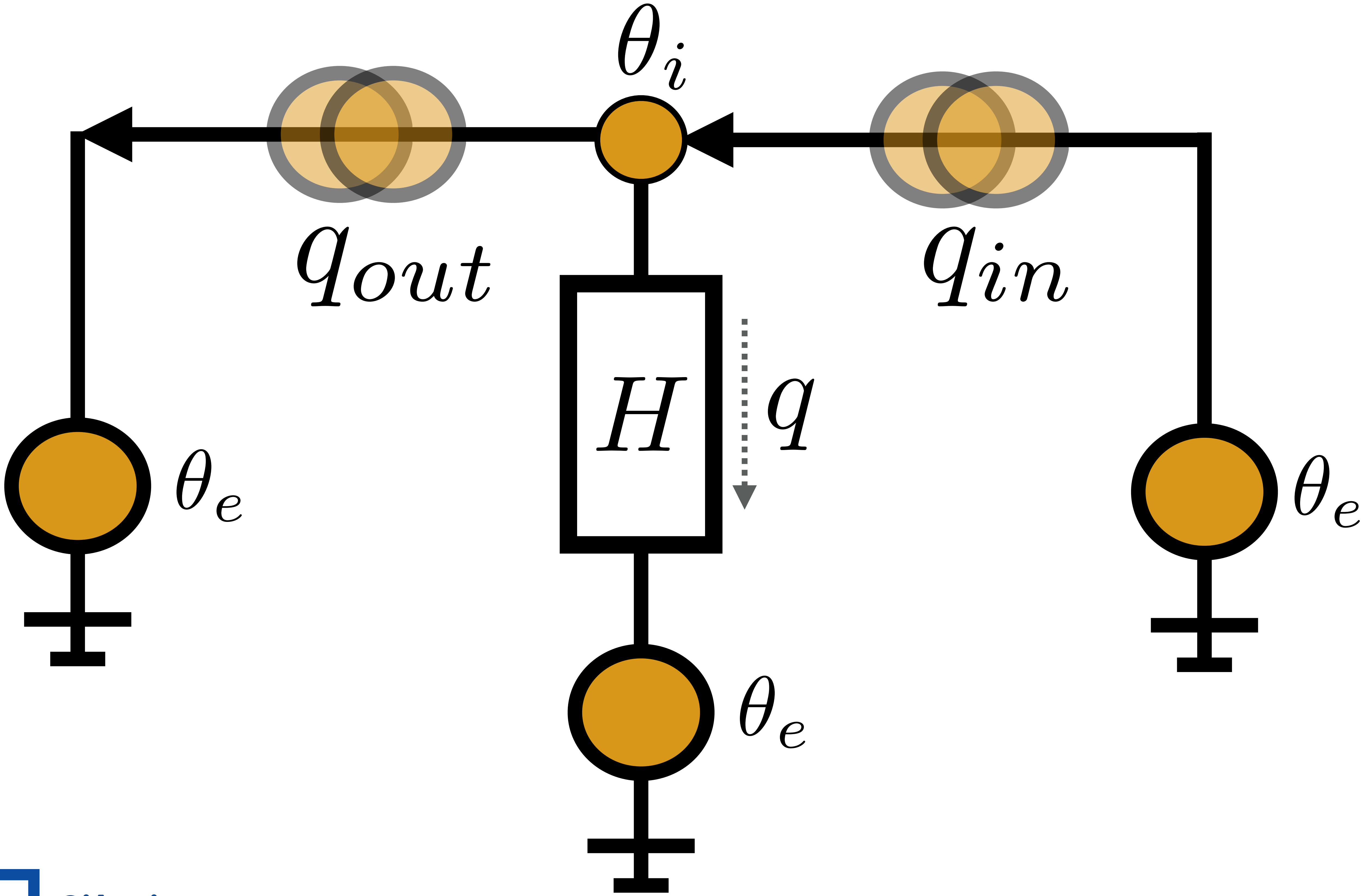

$q_{out} = \dot{m}c\theta_i$
θ_i
θ_e
$q_{in} = \dot{m}c\theta_e$



Ciências ULisboa



H overall thermal transmittance



Ciências ULisboa

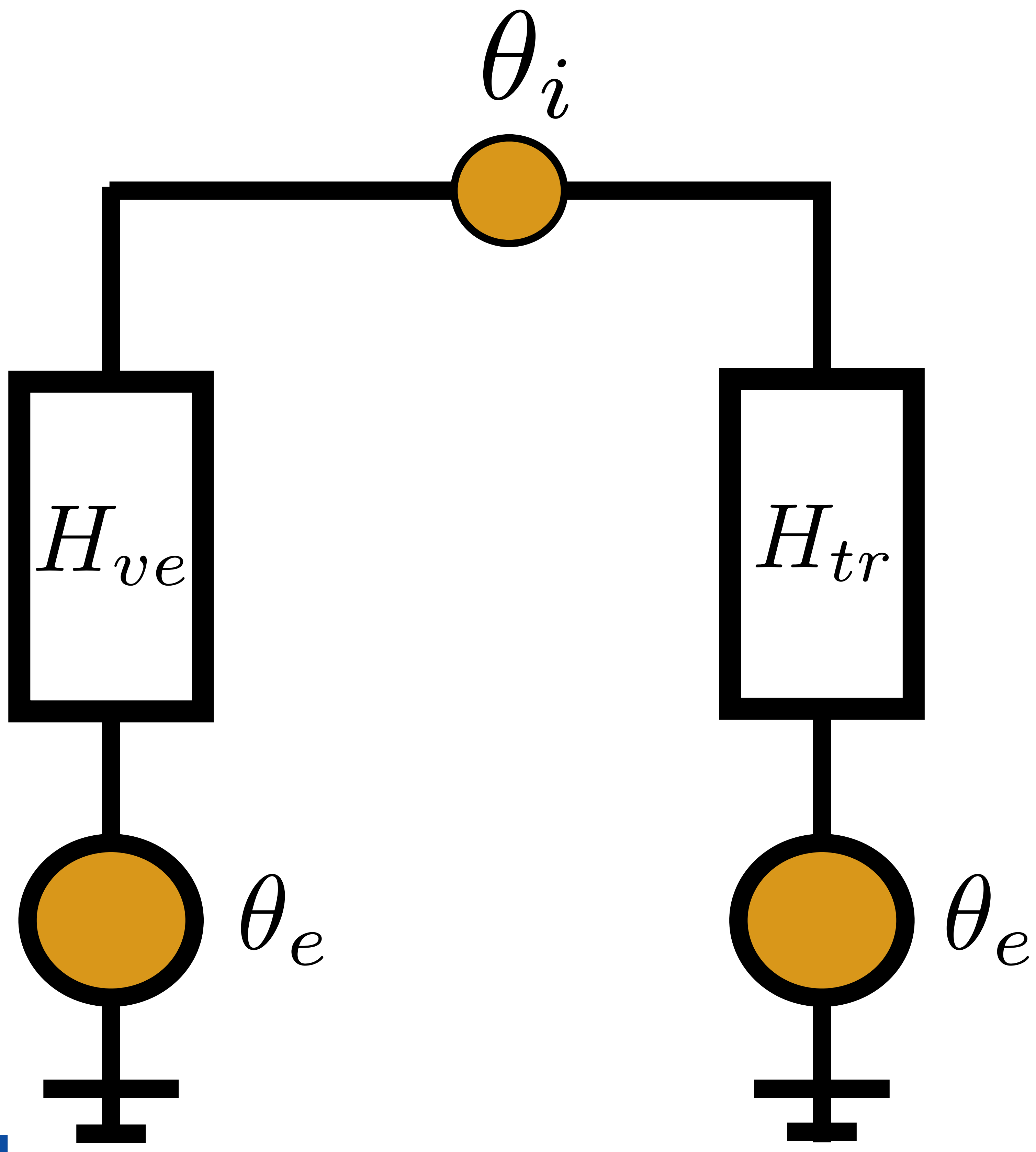
room air temperature node

$$q_{in} = q_{out} + q$$

$$\dot{m}c\theta_e = \dot{m}c\theta_i + H(\theta_i - \theta_e)$$

$$\dot{m}c(\theta_e - \theta_i) = H(\theta_i - \theta_e)$$

$$H_{ve}(\theta_e - \theta_i) = H_{tr}(\theta_i - \theta_e)$$



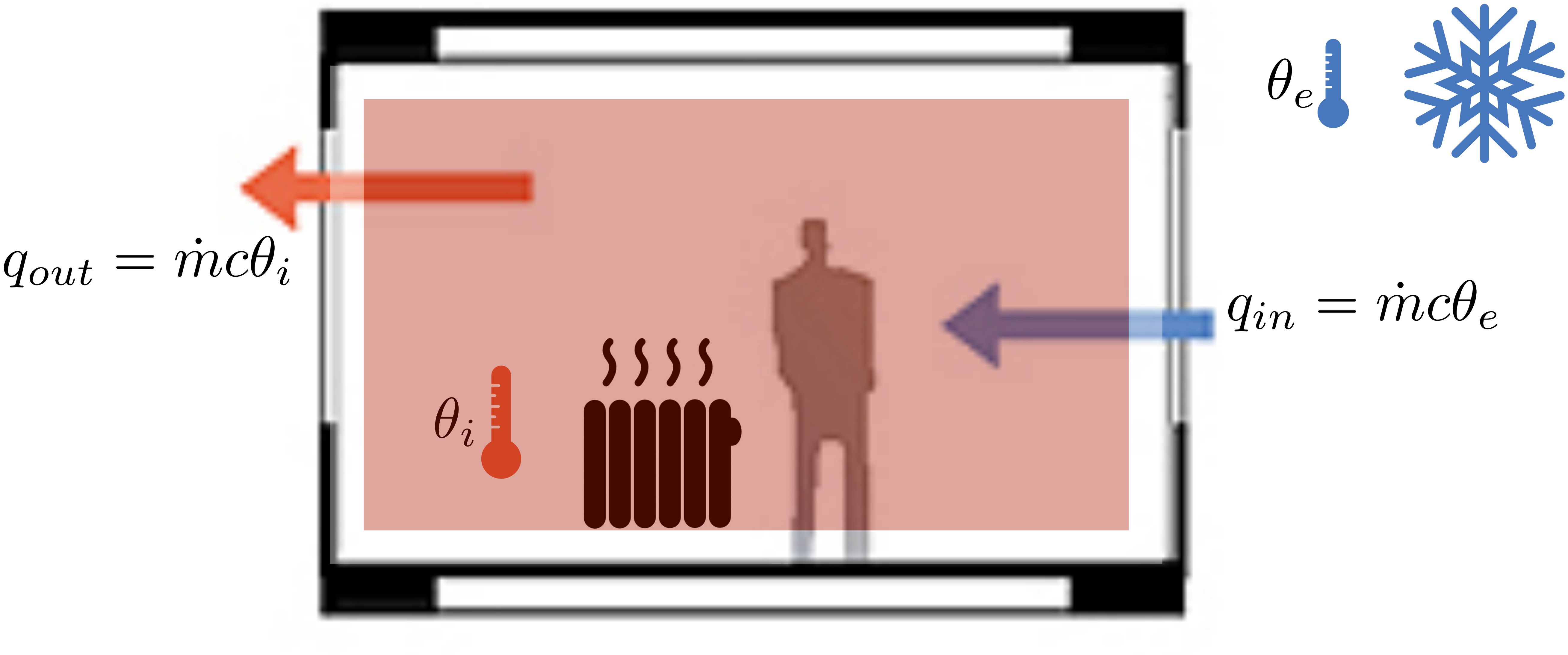
H_{ve}

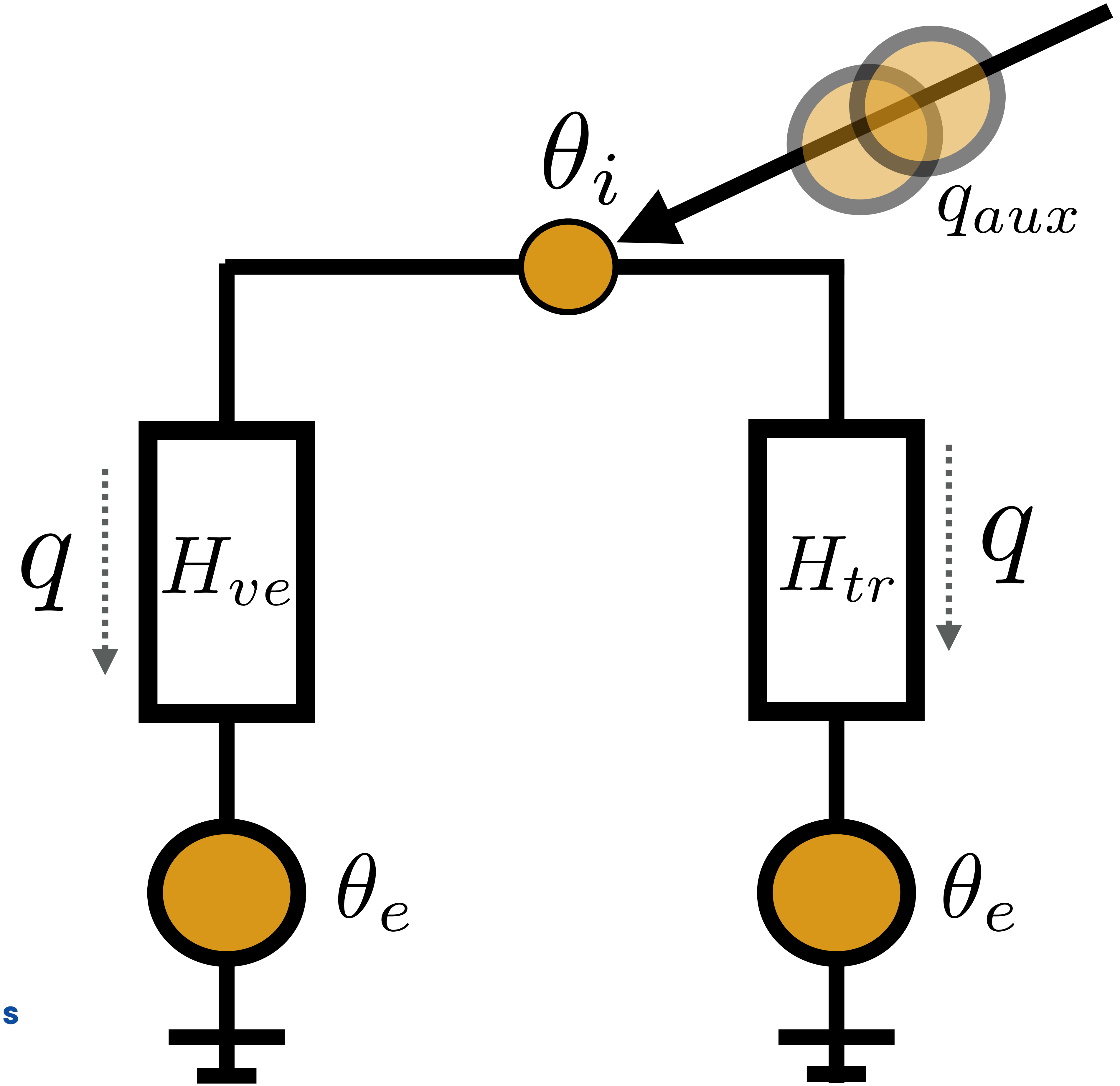
Ventilation thermal conductance

H_{tr}

Transmission thermal conductance

Ciências ULisboa

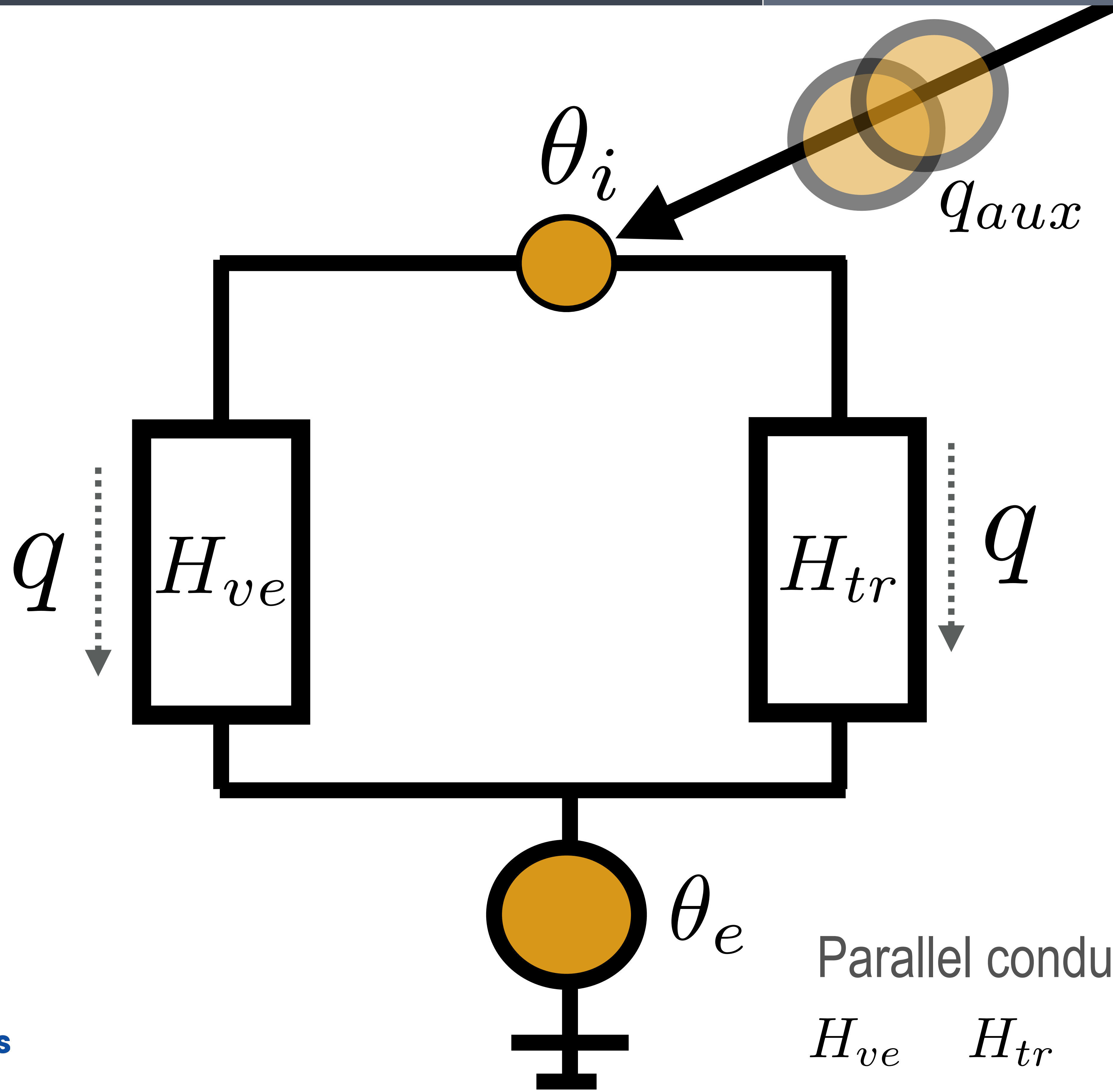

$q_{out} = \dot{m}c\theta_i$
θ_i
$q_{in} = \dot{m}c\theta_e$
θ_e


θ_i
q_{aux}
q
H_{ve}
H_{tr}
q
θ_e
θ_e

energy balance at θ_i

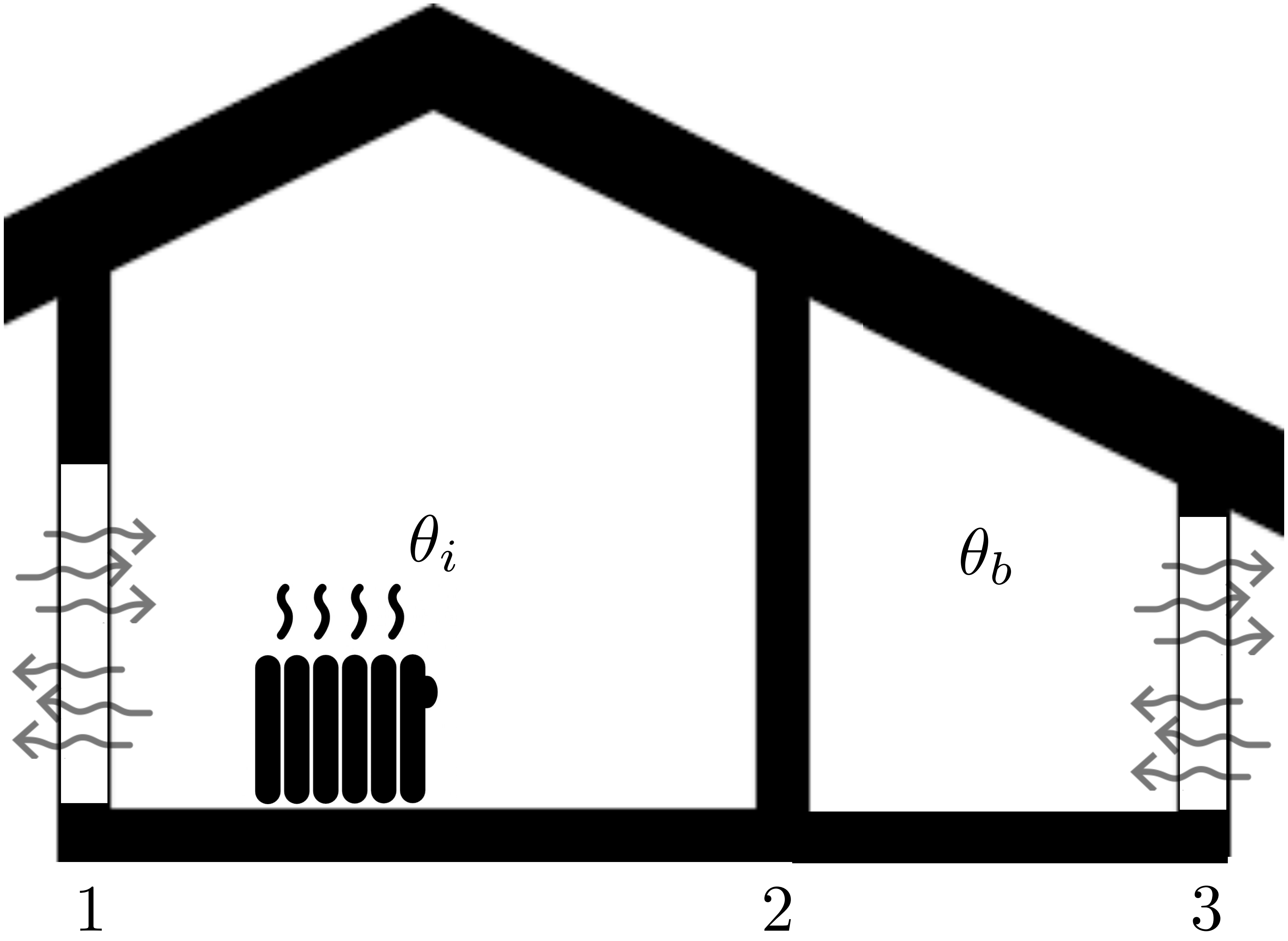
$$q_{aux} = H_{ve}(\theta_i - \theta_e) + H_{tr}(\theta_i - \theta_e)$$

$$q_{aux} = (H_{ve} + H_{tr})(\theta_i - \theta_e)$$

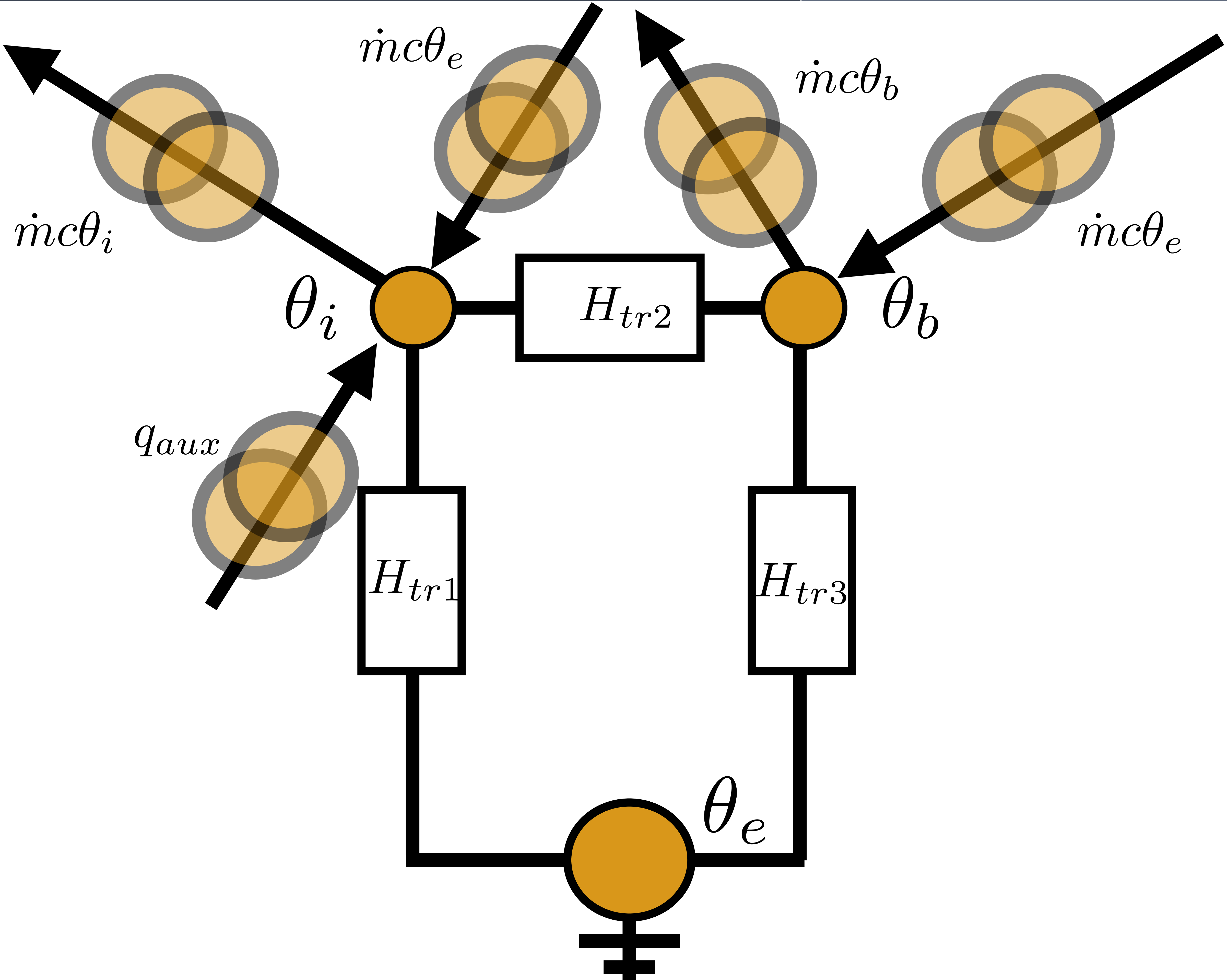


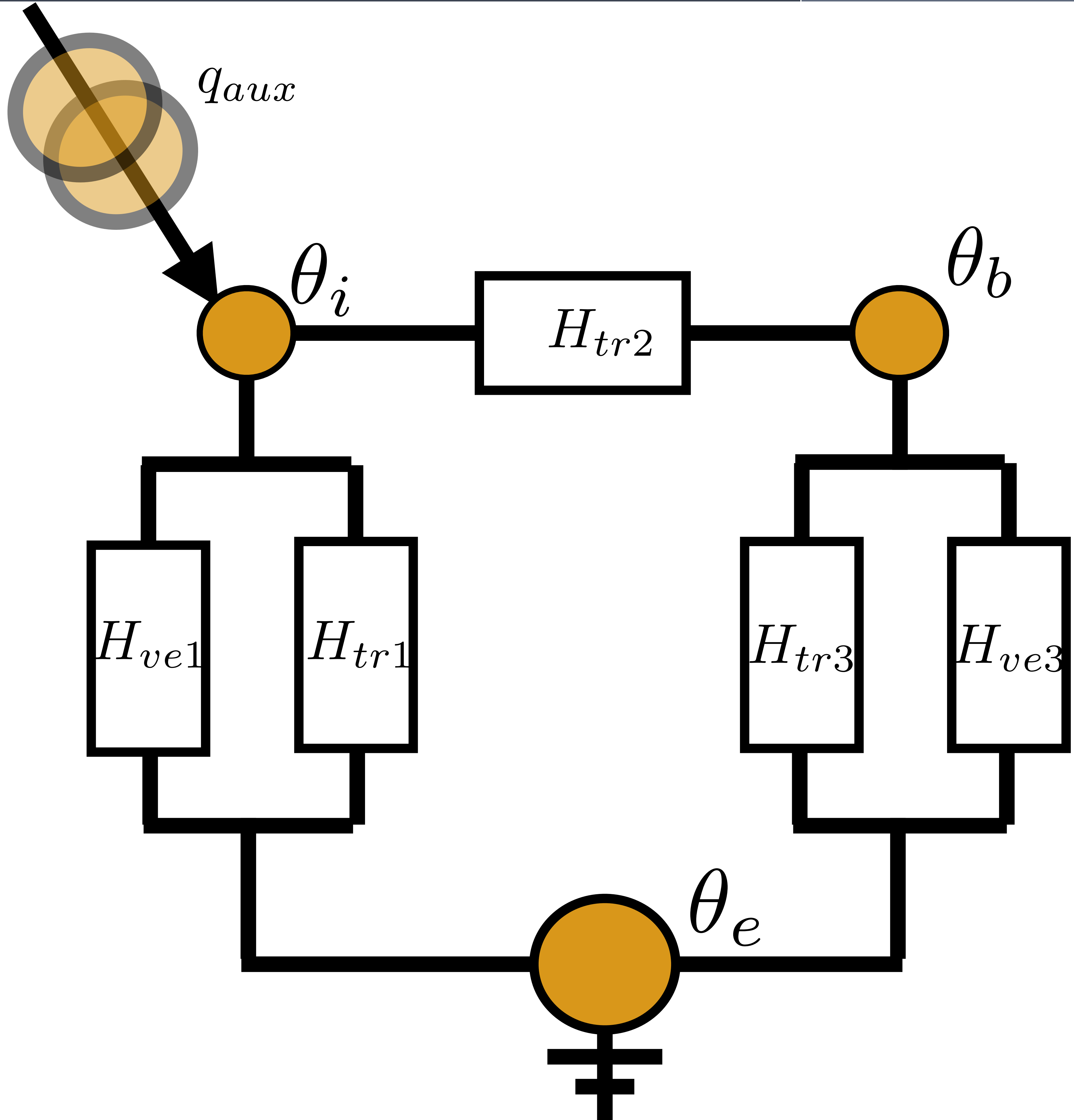
Parallel conductances

H_{ve} H_{tr}



Ciências ULisboa

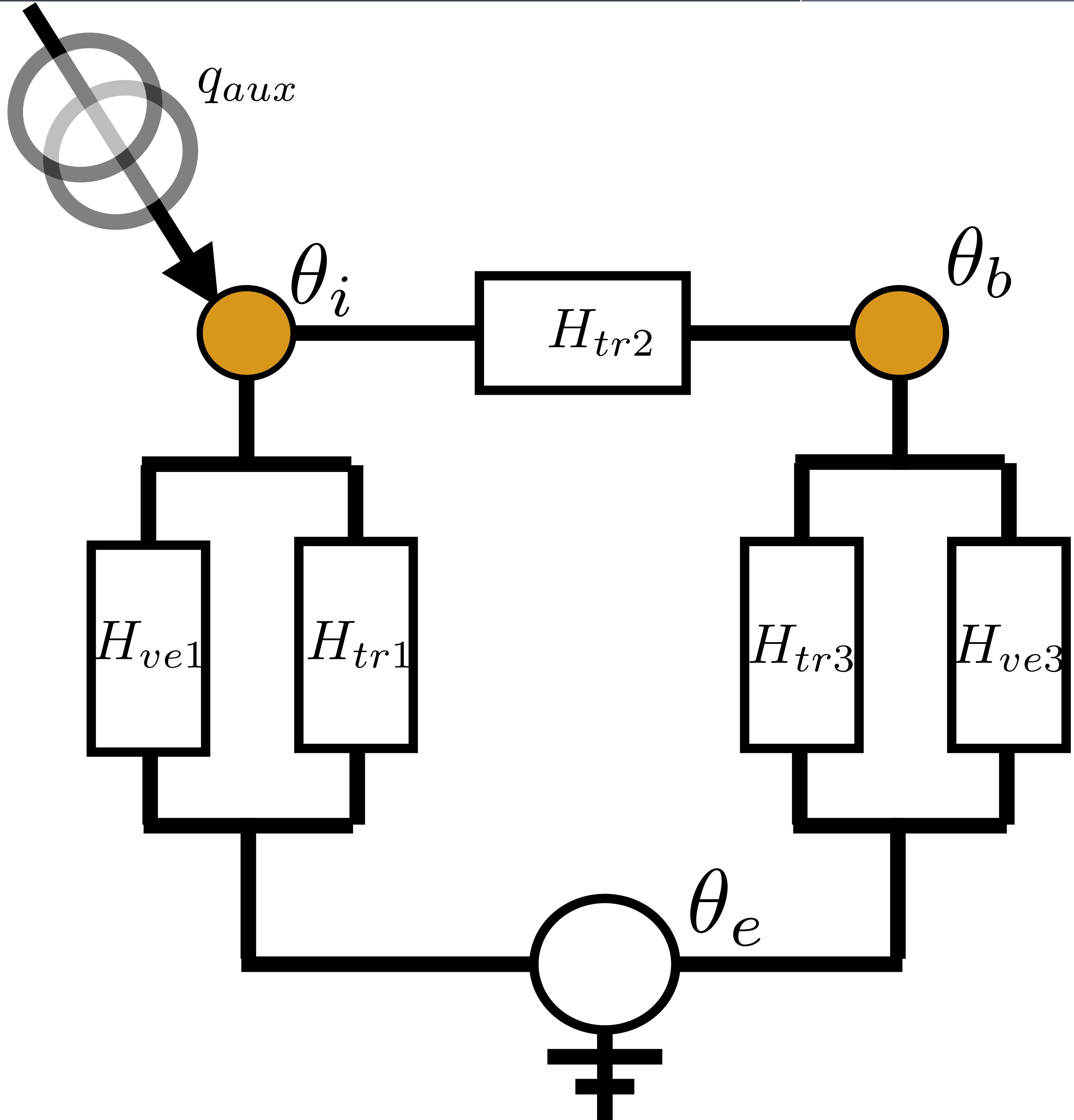

$\dot{m}c\theta_e$
$\dot{m}c\theta_b$
$\dot{m}c\theta_i$
$\dot{m}c\theta_e$
θ_i
H_{tr2}
θ_b
q_{aux}
H_{tr1}
H_{tr3}
θ_e


q_{aux}
θ_i
H_{tr2}
θ_b
H_{ve1}
H_{tr1}
H_{tr3}
H_{ve3}
θ_e

two equations → energy balance at θ_i

two equations → energy balance at θ_b

at each node: heat flow in = heat flow out

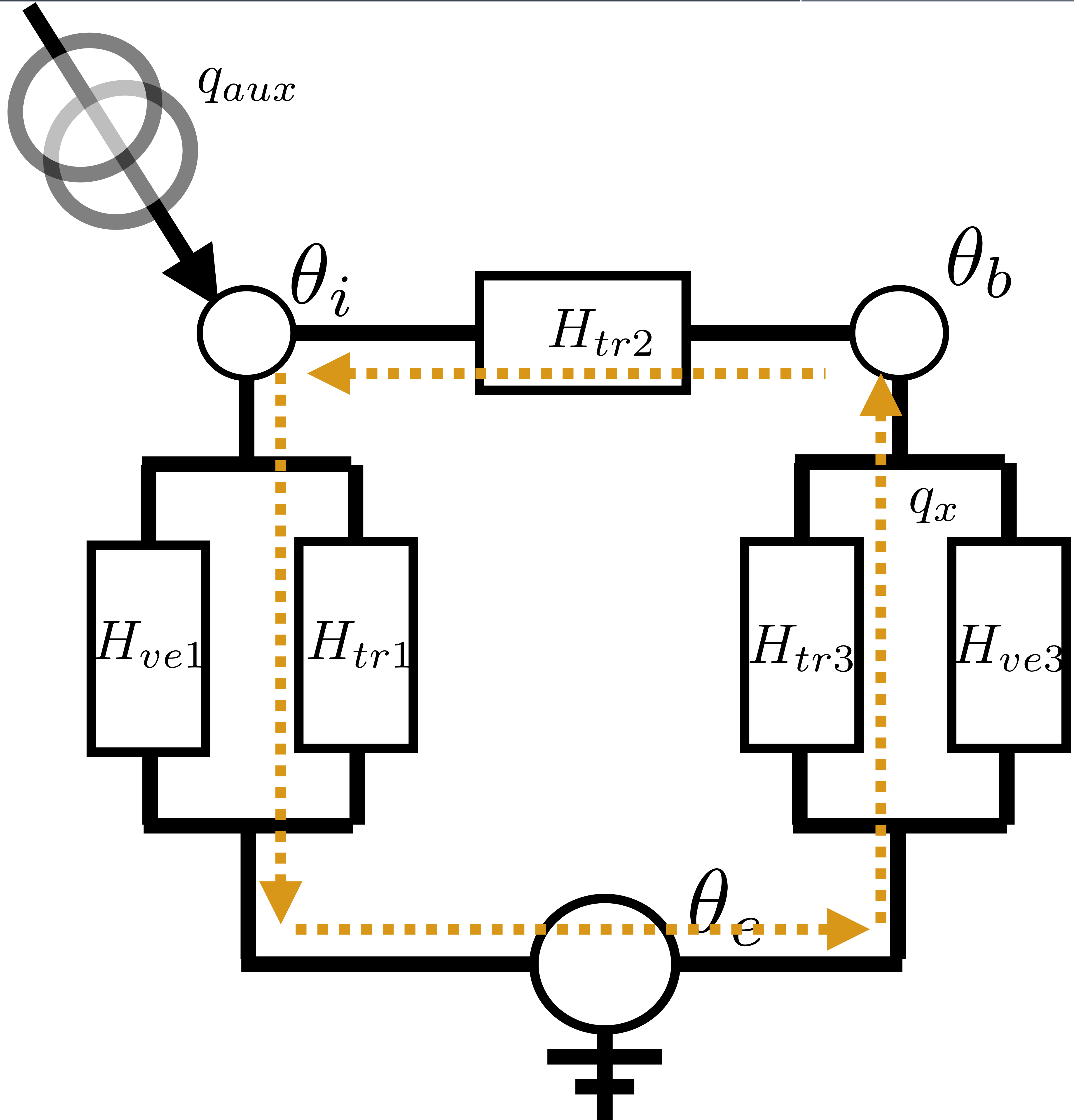

q_{aux}
θ_i
H_{tr2}
θ_b
H_{ve1}
H_{tr1}
H_{tr3}
H_{ve3}
θ_e

for each closed loop

$$\sum_i \frac{q_i}{H_i} = 0$$

for an open loop

$$\sum_i \frac{q_i}{H_i} = \Delta\theta$$

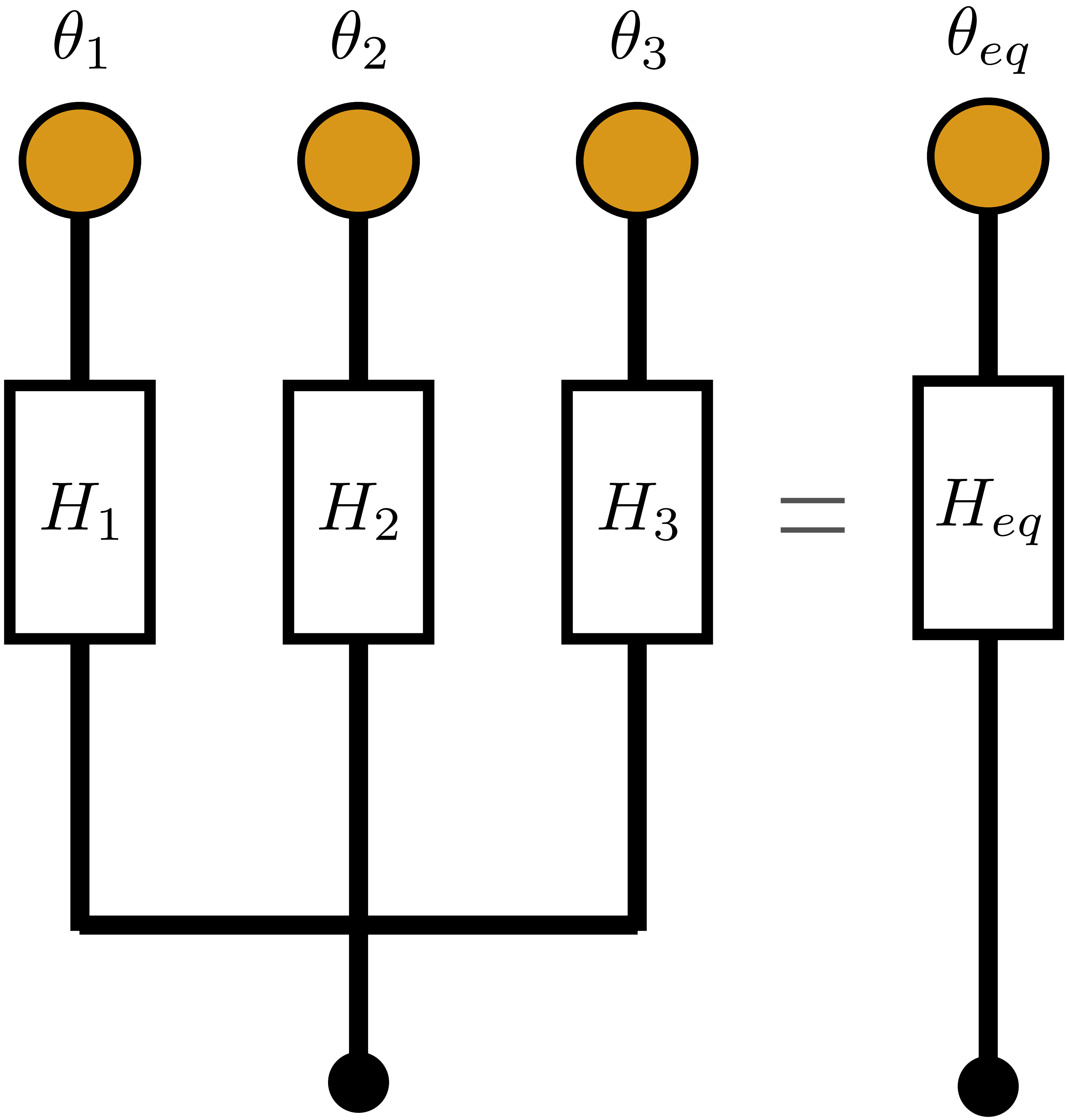

q_{aux}
θ_i
H_{tr2}
θ_b
q_x
H_{ve1}
H_{tr1}
H_{tr3}
H_{ve3}
θ_e

MESH

- Less equations
- Appropriate for analytical resolution

NODAL

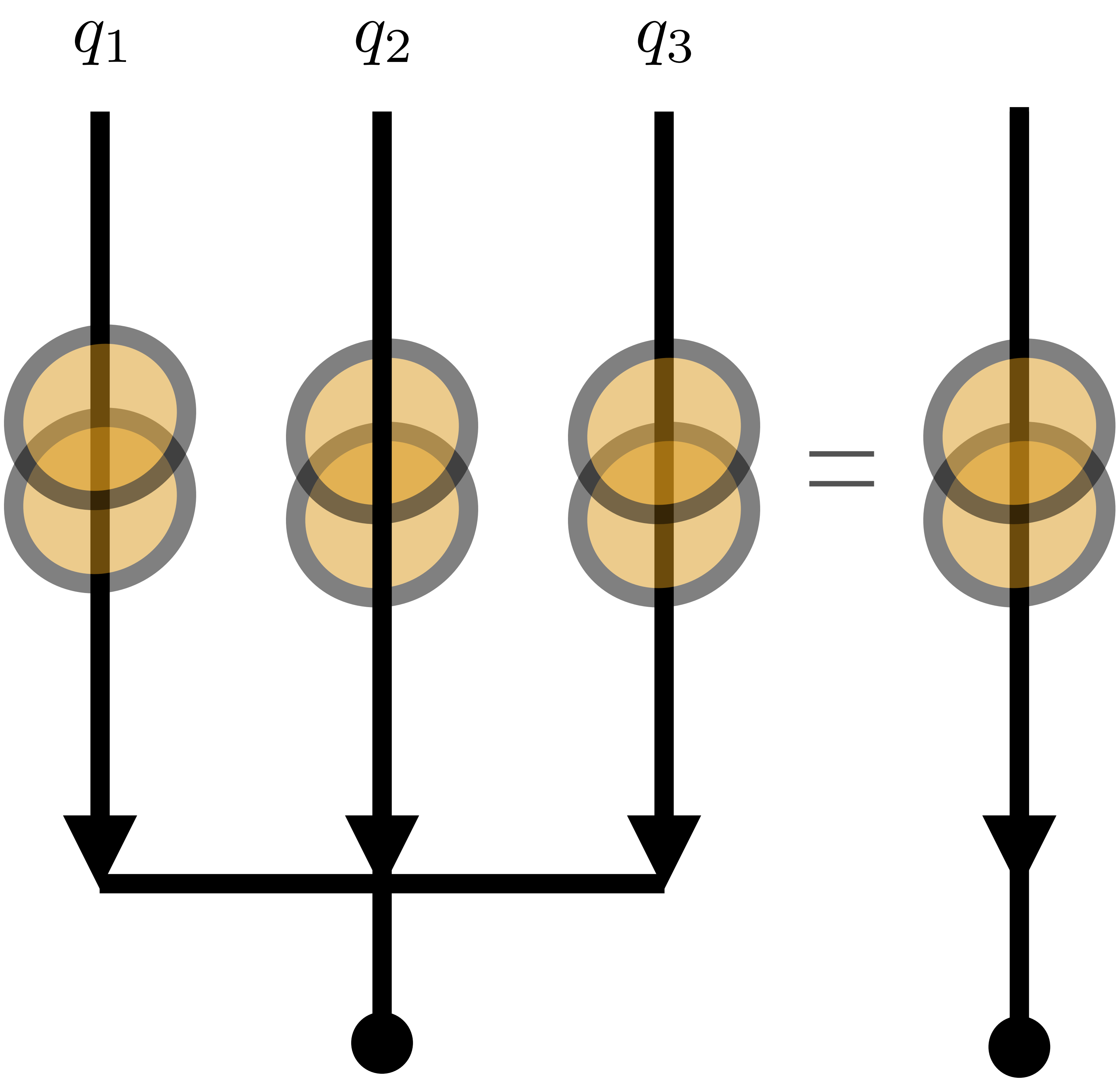
- One equation for each unknown temperature node
- Appropriate for numerical matricial resolution



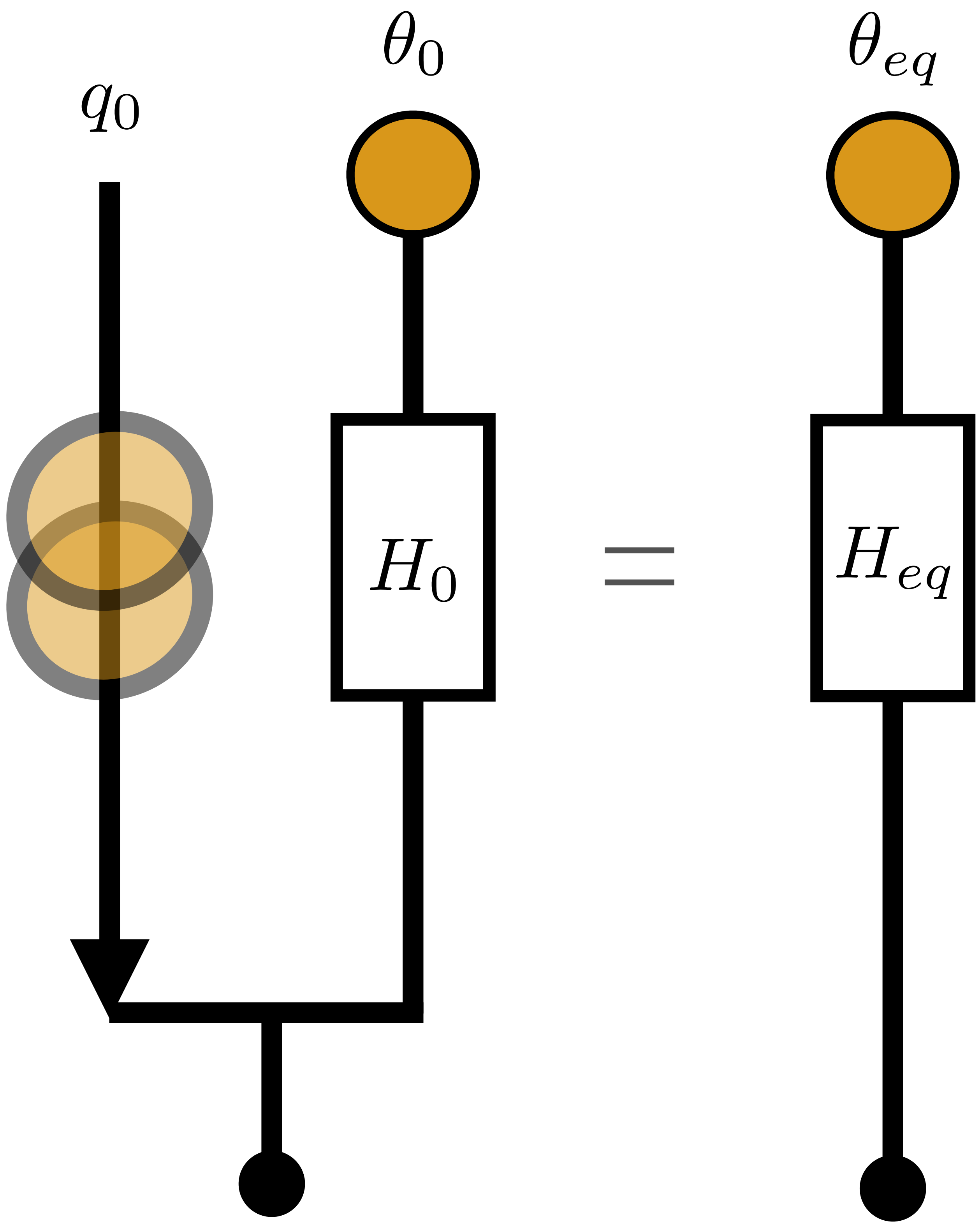
$$\theta_{eq} = \frac{H_1\theta_1 + H_2\theta_2 + H_3\theta_3}{H_1 + H_2 + H_3}$$

$$H_{eq} = H_1 + H_2 + H_3$$

Ciências ULisboa

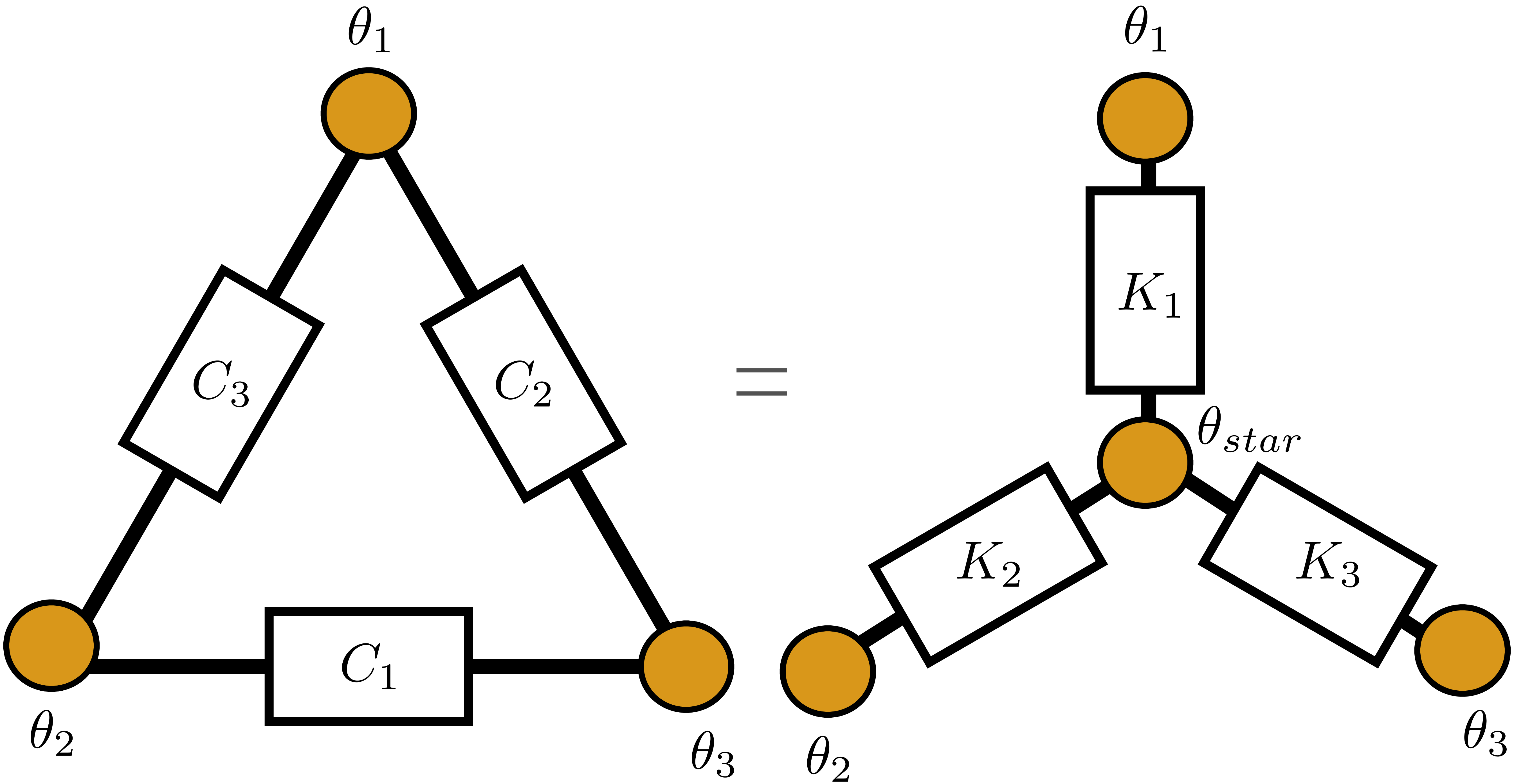


$$q_{eq} = q_1 + q_2 + q_3$$



$$\theta_{eq} = \theta_0 + \frac{q_0}{H_0}$$

$$H_{eq} = H_0$$



$$C_1K_1 = C_2K_2 = C_3K_3 = C_1C_2 + C_2C_3 + C_3C_1 = \frac{K_1K_2K_3}{K_1 + K_2 + K_3}$$

$$\theta_{star} = \frac{K_1\theta_1 + K_2\theta_2 + K_3\theta_3}{K_1 + K_2 + K_3}$$